**CS206** **October 21, 2000**

# TEST 1

**Instructions:** Do all your work in the blue exam books. Please write your answers IN THE GIVEN ORDER, though you may solve problems in any order. There is no need to reduce answers to simplest terms. You may use books and notes, but all work must be your own. Show *ALL* your work. You will get *little* or *no* credit for an unexplained answer. The value of each question appears in parentheses. Use this as a guide in allocating your time. An asterisk (*) denotes a more challenging question. There are 70 points and 10 (*) points. 70 should be a good score.

1. In the "odd poker" experiment $\mathcal{E}$ you randomly pick a card from a standard deck, record what you got, replace it, and then repeat these steps four more times (i.e., an ordered sample of 5 cards).

   (a) (5 pts) Carefully describe the sample space $S$ and find $|S|$, its size. Explain your answer.

   (b) (5 pts) Find the probability of $A = \{$all 5 cards are Kings$\}$. Explain. (Use equally likely probability measure $P$ throughout this problem.)

   (c) (5 pts) Find the probability of $B = \{$all 5 cards are the same value (i.e., ignore suits)$\}$. Explain your answer.

   (d) (7 pts) Find $P_B(A)$, the conditional probability of $A$ given $B$. Explain. Are $A$ and $B$ independent? Explain.

   (e) (7 pts) Find the probability of $D = \{$the second card is a higher value than the first card$\}$. Explain.

   (f) (8 pts) Find the probability of a full house (3 cards of the same value (say $x$) and the other two both of another value $y \neq x$).

   (g) (8 pts) Find the probability of one pair (2 cards have the same value, say $x$, and the other three have values different from $x$ which are also different from each other).

2. The parts of this question are not related.

   (a) (8 pts) There are twelve people waiting for an elevator to the top floor. Four are too thin, four are too fat, and four are "just right". The elevator can only take four people at a time. The experiment is to choose the four to go in the first trip, the four for the second trip, and the last four who will go in the third trip. What is the probability that the thin ones all travel together? Carefully explain.

   (b) (7 pts) If you randomly choose a binary tree with 4 nodes, what is the probability that it has 2 leaves (nodes with NO children)? Explain.

   (c) (10 pts) In bridge, what is the probability that all four hands have an ace and a king?

   (d) (*) (10 pts) In the 6 hat experiment, what is the probability that none of the first three people get their own hats?