

TEST 1

Instructions:

- You may use *ONE* page of prepared notes (both sides), but otherwise the test is *closed book*. All work must be your own.
 - Show *ALL* your work. You will get *little* or *no* credit for an unexplained answer.
 - Do all your work in the blue exam books. Please **WRITE** your answers **IN THE GIVEN ORDER**, though you may **SOLVE** problems in **ANY** order.
 - The value of each question appears in parentheses. Use this as a guide in allocating your time. There are 75 points, roughly a point a minute
 - There is no need to reduce answers to simplest terms.
1. (5 pts) Write down your student number. What is the value of X , the *second smallest* digit? What is the value of Y , the maximum digit? Now show how to compute the value of $(X + 5)_3$. Then show how to compute the value of the binomial coefficient $\binom{Y+7}{X+4}$.
 2. In successive semesters a student takes CS111, CS112, CS113, CS205, CS206, CS344, six of the required courses for a Computer Science major. In each course she receives one of the grades (from best to worst) A, B^+, B, C^+, C, D, F . The grades are given at random.
 - (a) (5 pts) Carefully describe the sample space S , and find its size, $|S|$.
 - (b) (5 pts) Find the probability (throughout use equally likely probability measure P) that all her grades are C or better. As always, explain your answer.
 - (c) (5 pts) Find the probability she gets the same grades in CS111, CS112, and CS113. Explain.
 - (d) (5 pts) Find the probability she gets exactly two B^+ grades. Explain.
 - (e) (5 pts) Find the probability that she gets at least two *different* grades in the sequence of six courses. Explain.
 - (f) (5 pts) Let C denote the event in part (c) and D the event in part (d). Are these events independent? Explain.
 - (g) (5 pts) Now find the probability that *neither* C nor D occur. Explain.
 - (h) (5 pts) Find the probability that she gets *exactly* two different grades. Explain.
 - (i) (10 pts) Find the probability of the event $G = \{\text{all grades are different}\}$. Then find the probability of the event $H = \{\text{her grades in the sequence CS111, CS112, CS113 increase}\}$. Explain.
 3. (5 pts) Give some constructive criticism of the course so far: (i) What is good, and should be continued? (ii) What is bad, and should be changed? (iii) What is missing, and should be added?

OVER

4. The following questions are not related.

- (a) (8 pts) Twelve students (three of them are cousins) are randomly split into three groups, 4 people in each group, for a game. What is the probability of A , the event that the cousins are all in the same group? Explain, describing S , the sample space you use, its size $|S|$, and $|A|$. Now find the probability they are all in different groups?
- (b) (7 pts) There are 27 students in CS206, and 11 of them women. 14 students are chosen at random to take their exam in a different room. What is the probability that *exactly* four women were chosen? What is the probability that *at least* four were chosen? Explain.