(Write up convincing answers to the questions marked by an asterisk (*), but not the unstarred questions. In fact some of the unstarred questions will be worked in the recitations. (**) is more challenging. If you are in the honors section, hand in solutions to (**) as well. Your solutions are due in class on September 21, 2015 (at the earliest).

1. (Sets) Decide whether the following statements are TRUE or FALSE and give a convincing argument to support your claim.
   
   a) (*) $(A \cup B) \setminus C = A \cup (B \setminus C)$.
   b) $(A \cup B) \setminus A = B$.
   c) (*) $(A \setminus B) \cup B = A$
   d) $(A \cup B) = (A \setminus B) \cup (B \setminus A) \cup (A \cap B)$.
   e) (*) $(A \cap B)c \setminus Ac = Bc$
   f) $\bigcap_{i=1}^{n} A_i \subseteq A_1$.
   g) (*) $A_1 \subseteq \bigcup_{i=1}^{n} A_i$.
   h) $\bigcup_{i=1}^{n} A_i \subseteq \bigcup_{i=1}^{n+m} A_i$, $m > 0$; i.e., $\bigcup_{i=1}^{k} A_i$ increases with $k$.
   i) (*) $\bigcap_{i=1}^{n} A_i \supseteq \bigcap_{i=1}^{n+m} A_i$, $m > 0$; i.e., $\bigcap_{i=1}^{k} A_i$ strictly decreases with $k$.

2. (NEW) Let $A, B$ and $C$ be events in a sample space $S$. Express the following events - described in English - using the operators $\cup$, $\cap$, $\setminus$, and complement.
   
   a) (*) $A$ occurs but neither $B$ nor $C$ do.
   b) At most two of the events occur.
   c) (*) Exactly two of the events occur.
   d) Exactly one of $A, B, C$ occur, but not $C$.

3. (Random Experiments and Sample Spaces) For each of the following random experiments, carefully describe the sample space, $S$. Try to compute $|S|$, the size of $S$, and explain your answer.
   
   a) (*) A coin is tossed 4 times.
   b) A die is thrown 3 times.
   c) (*) Five people enter the elevator in the basement of a building with 3 floors. Each states where he will get out.
   d) A box has 10 chips numbered 1 through 10. A chip is drawn from the box, its value noted, and then it is returned to the box. Then a second chip is drawn and its value noted.
   e) As in (d) but the chips are NOT returned to the box.
   f) (*) A die is tossed. If it shows an EVEN face, a coin is thrown. Otherwise (the die showed an ODD face), the die is thrown again and the results are written down.
   g) A 4 node, rooted binary tree is written down.
   h) (***) The hatcheck experiment with $n = 4$ people is performed (i.e., the hats of the 4 are randomly permuted, or redistributed, one hat to each person). Write down the sample space. How many of the outcomes are derangements (nobody gets their own hat)?

4. (Events) Carefully describe the events $A, B, A \cup B$, and $A \cap B$ in the following sample spaces from 3, above, and determine the sizes of these events (preview to COUNTING).
(a) (*) In 3a, \( A = \{\text{Head on the first and last tosses}\} \), \( B = \{\text{at least 2 tails}\} \).
(b) In 3b, \( A = \{\text{at least one even-score face}\} \), \( B = \{\text{all faces 4 or more}\} \).
(c) In 3d, \( A = \{\text{same chip both times}\} \), \( B = \{\text{chip 10 is not chosen}\} \).
(d) (*) In 3f, \( A = \{\text{the coin is NOT thrown}\} \), \( B = \{\text{exactly one dice throw showed a four}\} \).
(e) (*) In 3g, \( A = \{\text{the root has two children}\} \), \( B = \{\text{the tree has height 4}\} \), the height being the number of edges on a longest path from the root to a leaf.
(f) (**) As in 3d, a box has 3 red and 5 black chips. In this experiment, you pick a chip, note its color, and return it to the box. The experiment continues until you have picked more red than black chips. (i) Describe the sample space and then let \( A = \{\text{stop on third pick}\} \), \( B = \{\text{stop before sixth pick}\} \).

5. (Probability) Do the following:

(a) Use equally likely probability measure and compute the probability of the events in un-starred parts of question 3, Homework 1.

(b) (*) \( A, B, C \) are events in a probability space \((S, P)\). If \( P(A) = 1/3 \), \( P(B) = 1/2 \), and \( P(A \cup B) = 3/4 \) compute \( P(A \cap B) \), \( P(A^c \cup B^c) \), and \( P(A^c \cap B) \).

(c) (*) An experiment has two outcomes, one with probability \( p \), the other with probability \( p^2 \). What is \( p^2 \)?

(d) Prove \( P[(A \cap B^c) \cup (A^c \cap B)] = P(A) + P(B) - 2P(A \cap B) \). (*) Describe this event in English.

(e) For the unstarred parts of question 4, and assuming that \( P \) is equally likely in the relevant experiments, find \( P_A(B) \), the conditional probability of \( B \), given \( A \).