CS206 **December 13, 1997**

TEST 2

Instructions: Do all your work in the blue exam books. Please write your answers IN THE GIVEN ORDER, though you may solve problems in any order. There is no need to reduce answers to simplest terms. You may use books and notes, but all work must be your own. Show *ALL* your work. You will get *little* or *no* credit for an unexplained answer. The value of each question appears in parentheses. Use this as a guide in allocating your time. An asterisk (*) denotes a more challenging question.

1. Experiment $\mathcal{E}$ is to toss two coins, a nickel and a dime. Its sample space is $S = \{HH, HT, TH, TT\}$ where, e.g., TH means "nickel is tail, dime is head". These coins have the property that the probability is 2/10 for HH, 3/10 for HT, 2/10 for TH, and 3/10 for TT.

   (a) (5 pts) Find the probability that the nickel is Head? Explain. Repeat for the probability that the dime is Head? Are either of the coins fair? Explain.

   (b) (10 pts) Let X_N be the indicator of the event that the nickel comes up Head and X_D the indicator of the event that the dime comes up Head. Are these random variables independent? Carefully explain your answer.

   (c) (10 pts) Let X be the random variable that counts the number of heads observed when $\mathcal{E}$ is performed. Find (i) Range(X), (ii) f_X, the frequncy function of X, (iii) $V(X)$, the variance of X.

   (d) (7 pts) Find the generating function $\phi_X(s)$. Use it to compute $E(X)$. Explain.

   (e) (10 pts) $\mathcal{E}^{(2)} = \mathcal{E}_1, \mathcal{E}_2$ is the experiment of performing $\mathcal{E}$ twice ($\mathcal{E}_i$ denotes the i^{th} performance), and $S^{(2)}$ is its sample space. Find the product probability that in $S^{(2)}$, NO head is observed. Explain. Repeat for the event that exactly TWO heads are observed.

   (f) (10 pts) In $S^{(2)}$ let Y count the number of nickel Heads observed and Z, the number of dime Heads. Find $Cov(Y, Z)$, the covariance of these random variables, and explain your answer.

   (g) (10 pts) $\mathcal{E}^{(10)} = \mathcal{E}_1, \ldots, \mathcal{E}_{10}$ is the experiment of performing $\mathcal{E}$ ten times. How many heads are expected? Carefully explain your answer.

   (h) *(8 pts) With $\mathcal{E}^{(10)}$, what is the probability that exactly 7 heads come up? Explain.

(OVER)

2. Your computer program has exactly 4 "bugs". The experiment is to conduct a debugging session. The outcomes are (i), ONE bug is removed with probability 2/3, and (ii), NO bugs are corrected (probability = 1/3). The experiment is repeated until the 4 bugs have been removed.

   (a) (8 pts) Carefully describe the sample space, S, of this experiment. What is $|S|$, its size.?

   (b) (7 pts) If you do three debugging sessions, what is the probability that at least 1 bug is removed? Explain.

   (c) (8 pts) Let X be the number of sessions needed to remove 2 bugs. What is the probability that $X = 4$? What is $E(X)$? Explain.

   (d) (5 pts) If you do 6 debugging sessions, how many bugs do you expect to remove? Explain.

   (e) (10 pts) If you are allowed a maximum of 6 debugging sessions, what is the probability that you will eliminate ALL bugs. Explain.

   (f) *(7 pts) You do 5 debugging sessions. What is the probability of the event that at least two bugs are removed and the second one needed MORE sessions for its removal than the first? Explain.

   (g) (15 pts) A programmer suspects that 2/3 may NOT be the true probability of correcting a bug. The data is a history of 1800 debugging sessions in which 1400 bugs were removed. Use Tchebycheff's inequality to help decide if the data support $\mathcal{P} = 2/3$.

3. A computer can write to disk (D), terminal (T), or printer (P). There are currently 5 jobs in the system and each requests ONE output device according to the probabilities 1/2 for the disk, 1/3 for the printer and 1/6 for the terminal. Use product probability on the sample space S.

   (a) (5 pts) Carefully describe S and give its size $|S|$.

   (b) (5 pts) Let X_T be the number of jobs requesting a terminal, X_D the number asking for the disk, and X_P, the number requesting the printer (so $X_D + X_T + X_P = 5$). Are these random variables pairwise independent? Explain.

   (c) (10 pts) Let N be the number of devices requested. Find the probability that $N = 3$.

   (d) *(10 pts) Find $E(N)$ and $V(N)$, the mean and variance.

4. (10 pts) Toss a fair coin 4 times. Let $X_i = 1$ if the i^{th} toss is Head or -1 if it is Tail. Write $S_0 = 0$ and $S_j = X_1 + \cdots + X_j$ for the "advantage" of Heads over Tails after the j^{th} toss. Find the probability that $-1 \le S_j \le 1$, for $j = 1, \ldots, 4$, and explain your computation.