Try to write up solutions and hand them in by Feb. 23 in class. You can talk with others about the problems, but I want to insist that you write up solutions entirely on your own. If you use a source like a book, paper, or webpages from the internet, carefully cite the sources, along with your solution. If you did discuss with others, please cite them in your solutions (maybe different people for different problems). Finally, when you hand in your solutions, write the pledge “I followed the rules for this assignment” and then sign your name.

1. In the “collection of problems” do 1, 2a, 4c, 6, 7a, 8a, and 9; i may later ask for 14. Please note the problem collection has been revised. You should be using version 1d from Feb. 7.

2. Given $S = \{P_1, \ldots, P_n\} \subseteq \mathbb{R}^2$, the diameter $\text{diam}(S) = \max_{i \neq j} d(P_i, P_j)$ is the distance between the furthest pair of points. In class we gave an $\Omega(n \log n)$ lower bound for DIAMETER by reduction to set disjointness as follows:

Let $A = \{a_1, \ldots, a_n\}$ and $B = \{b_1, \ldots, b_n\}$ be the two sets whose disjointness will be tested. First, in $O(n)$ time, we map the elements of $A$ and $B$ to points on the circumference of the unit circle; the elements of $A$ will map to points in the first quadrant, and those of $B$ will map to points in the third. To do this we first compute

$$m = \min(a_1, \ldots, a_n, b_1, \ldots, b_n) \quad \text{and} \quad M = \max(a_1, \ldots, a_n, b_1, \ldots, b_n)$$

and then map the sets $A$ and $B$ to $\Theta = \{\theta_1, \ldots, \theta_n\}$ and $\Phi = \{\phi_1, \ldots, \phi_n\}$ (respectively) using

$$\theta_i \leftarrow \left(\frac{a_i - m}{M - n}\right) \frac{\pi}{2}; \quad \phi_i \leftarrow \pi + \left(\frac{b_i - m}{M - n}\right) \frac{\pi}{2}$$

for $i = 1, \ldots, n$. Finally we get the sets $S = \{s_1, \ldots, s_n\}$ and $T = \{t_1, \ldots, t_n\}$ for set-disjointness by

$$s_i \leftarrow (\cos \theta_i, \sin \theta_i) \quad \text{and} \quad t_i \leftarrow (\cos \phi_i, \sin \phi_i), i = 1, \ldots, n.$$

(a) Show that $S \cap T = \emptyset \iff \text{diam}(S \cup T) = 2$.

(b) Show you can get $S$ and $T$ from $A$ and $B$ in $O(n)$ (assume you have $\pi$ freely available and that you can compute $\sin$ and $\cos$, each in constant time).

(c) Carefully argue an $\Omega(n \log n)$ lower bound for diameter by reduction to set disjointness.
(d) (*) The reduction above is flawed because it asks us to compute $\pi, \sin, \cos$, features that are outside the unit cost RAM. Try to make the same reduction argument, but now using only operations that ARE allowed in the RAM model.

**HINT:** The key is to use trig functions and (rational) identities that relate them.

To start we map the sets $A$ and $B$ to angles $\Theta' = \{\theta'_1, \ldots, \theta'_n\}$ and $\Phi' = \{\phi'_1, \ldots, \phi'_n\}$ (respectively) now using

$$\theta'_i \leftarrow 2 \cdot \arctan \left( \frac{a_i - m}{M - n} \right) \in [0, \pi/2]; \quad \phi'_i \leftarrow 2 \cdot \arctan \left( \frac{b_i - m}{M - n} \right) \in [\pi, 3\pi/2]$$

for $i = 1, \ldots, n$. The points $s'_i = (\cos(\theta'_i), \sin(\theta'_i))$ and $t'_i = (\cos(\phi'_i), \sin(\phi'_i))$ can be shown to be rational functions of the data and accessible in the RAM model.

Carefully show how to do it, and if you don’t mind - say whether you came up with this idea or something like it all by yourself.

3. (*) Try 2b. The asterisk (*) here (and in 2d) above means “more challenging”. It also means you don’t have to do this. But if you do it and want your solution checked, hand it in; I won’t count it for grades (except possibly a small amount at the end).