Example: Solve by LU decomposition with complete pivoting:

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & 2 & 2 \\ -2 & 4 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$$

1. $PAQ = LU$ decomposition of coefficient matrix...

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & 2 & 2 \\ -2 & 4 & 2 \end{pmatrix} \xrightarrow[\text{col } 1 \leftrightarrow \text{col } 2]{\text{row } 1 \leftrightarrow \text{row } 3} > \begin{pmatrix} 4 & -2 & 2 \\ 2 & 0 & 2 \\ -2 & 1 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 4 & -2 & 2 \\ (\frac{1}{2}) & 1 & 1 \\ (-\frac{1}{2}) & 0 & 2 \end{pmatrix}$$

$$\cdots \quad \xrightarrow[\text{col } 2 \leftrightarrow \text{col } 3]{\text{row } 2 \leftrightarrow \text{row } 3} > \begin{pmatrix} 4 & 2 & -2 \\ (-\frac{1}{2}) & 2 & 0 \\ (\frac{1}{2}) & 1 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 4 & 2 & -2 \\ (-\frac{1}{2}) & 2 & 0 \\ (\frac{1}{2}) & (\frac{1}{2}) & 1 \end{pmatrix} = [L\backslash U]$$

$$I \xrightarrow{\text{row } 1 \leftrightarrow \text{row } 3} > \xrightarrow{\text{row } 2 \leftrightarrow \text{row } 3} > \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = P$$

$$I \xrightarrow{\text{col } 1 \leftrightarrow \text{col } 2} > \xrightarrow{\text{col } 2 \leftrightarrow \text{col } 3} > \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = Q$$

2. Solution of given system...

$$b' = Pb : \quad \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \xrightarrow{\text{row } 1 \leftrightarrow \text{row } 3} > \xrightarrow{\text{row } 2 \leftrightarrow \text{row } 3} > \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = b'$$

$$Ly = b' : \quad \left(\begin{array}{ccc|c} 1 & & & 2 \\ -\frac{1}{2} & 1 & & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 & 4 \end{array}\right) \implies y = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

$$Uz = y : \quad \left(\begin{array}{ccc|c} 4 & 2 & -2 & 2 \\ & 2 & 0 & 2 \\ & & 1 & 2 \end{array}\right) \implies z = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$x = Qz : \quad \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \xrightarrow{\text{row } 2 \leftrightarrow \text{row } 3} > \xrightarrow{\text{row } 1 \leftrightarrow \text{row } 2} > \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = x$$