Functional Programming With LISP (Scheme)

In **pure functional programming** a program is viewed **as a function** which can be called with a variety of **actual parameters** such a call constitutes **the input for this run** of the program . It is built by **defining it in terms of other functions** which are themselves defined in terms of functions etc. until, at bottom, function definitions are based on a few Data-Structures and built in functions. In LISP and Scheme, the main the data-structure is the List- ex. '(a b c 23 5). Basic built in commands can pick out the first member of a List (*car*) , the List that remains when the first member is removed (*cdr*), and a List constructor which can extend a List by a new firstmember (*cons*).

In **pure functional programming t**he functions are all **call by value**; there are no side-effects, **every function returns a value.** Throughout one uses **prefix notation within parenthesis ex. (f a b c).** There are no assignment statements. The only "variables" are formal parameters. In the early versions, of such languages, Lisp and Scheme, there is **no provision for data type declarations,** and a **variable may take any type** as long as the operations with which it is associated make sense for that type. **(**Elegant data structure definition facilities are found in functional languages developed later, ex. ML),

The management of storage is implicit-done by the interpreter. Also, typically, **functions are first class values,** that is**: a function can be the argument of a function, and a function can be returned by a function.** Furthermore function definitions within a function definition is allowed, so calling a function results in defining a new function. This brings in a question of scoping - in Scheme **scoping is static (lexical)** (depends on the initial program nesting) **(depends on the call),** while in older Lisp's it is **dynamic(depends on the call).**

Conceptually such a language is simple, uniform, elegant and of course has the full expressive power of any language. Without assignments however the same function on the same arguments will have to be repeated when needed at more than one place in the same definition (SCHEME has a **let** instruction which serves for limited form of assignments.. Without typing the compiler (if there is one) has no opportunity to catch many errors which will only become apparent at run time. A relatively minor problem is the large number of parenthesis require in Lisp and Scheme, for example, to define a function to return the reverse of the List given as a parameter requires about 20 parenthesis as shown.

{ define** (reverse L) (**cond** [(**null**? L) ()) [**else** (**cons** (lastmember L) (reverse(headlist L)))]) **}

Different types of parenthesis are used here to help differentiate what is delineated.

We only present a limited subset of the commands in Scheme since we we are focussing on its pure functional aspects ..

Although compilers exist for this language, an interpreter handles our version. The execution model to keep in mind is the ***(print (eval (read)))*** loop in the interpreter; it reads an **S-expression**, evaluates it , and then prints the result. In **evaluating an S-expression,** ***(e_1 e_2 e_3 ... e_k)***, a sequence of events occur:

(1) **evaluation of e_1** to get **the name of a function** (often evaluation is trivial because the name of the function is given explicitly.)

(2) **evaluation of e_2, e_3, ... e_k** to get **values of the arguments** of this function.

(3) **apply the function to these values**.

Example

*(**cons**(**lastmember** L) (reverse(headlist L)))*

e_1= cons

e_2= *(lastmember L)*]

e_{21}= lastmember

e_{22}= L

*e_3=(**revers**e(**headlis**t L))*

*e_{31}=**revers**e*

*e_{31}=(**headlis**t L) etc.*

An obstacle for beginning Scheme programmers is remembering that this evaluation takes place. For example, typing *(1 2)* at the Scheme prompt would result in an error because the system would not have a function named *1*. We can inhibit this evaluation in Scheme by **quoting the object.** For example, typing ***'****(1 2)* which literally represents the list whose first member is 1 and second member 2 would have resulted in Scheme writing back to us the list *(1 2)***.**

© 2006 M C Paull

1 Some **Primitive (Builtin) Operations**

The List is SCHEMES Major Data Structure.

A list is represented by (l_1 l_2 .... l_n) where l_i can be list or an element (symbol, boolean, number, function, or itself a list) ex (it seems that) (it (seems that) you)

A symbol, or a list may be interpreted literally or as variable or function names-quote '(...) makes the difference .

The functions car and cdr are used to get at elements or sublists of a list.

car is used to extract the first element of a list; that is, *(car '(1 2))* is (returns) *1* and *(car '((1) 2))* is *(1)*. The difference between these two lists *(1 2)* and *((1) 2)* is that the first one has two elements, the atoms *1* and *2*, whereas the second one has two elements, the list *(1)* and the atom *2*.

cdr is used to get at the rest of the list, leftover" after a *car* operation. Therefore, *(cdr '(1 2))* is the list *(2)* and *(cdr '((1) 2))* is the list *(2)* as well. Note that after we take *car* or *cdr* of a list that list remains unchanged; nothing is destroyed in it. This is sometimes called copy semantics". These functions are merely a way of delineating sub-lists within a list.

These two functions can be nested; for example,

(car (cdr '(1 (2 3) 4))) = (2 3) because *(cdr '(1 (2 3) 4)) = ((2 3) 4)* whose *car* is *(2 3)*. One can write this combined function in shorthand notation as ***(cadr*** *'(1 (2 3) 4))*, thereby using the \d" and \a" to indicate the order of application of the *car*'s and *cdr*'s. *cons* is used to construct lists from an element and a list (i.e., a *car* and a *cdr*). Thus, *(cons 1 '(2 3))* is *(1 2 3)* and *(cons '(1) '(2 3)) is ((1) 2 3).*

Booleans

Unary

null? is a unary predicate ***ex. (null? E)*** which **returns** #t if its argument, ***E***, is *NIL* (i.e., empty list () or the atom *NIL*) and *NIL* otherwise.

zero? is a unary predicate which returns #t if its argument **has value 0** and *NIL* otherwise. integer? is a unary predicate which returns true if its argument is an integer and *NIL* otherwise.

list? is a unary predicate that returns true if its argument **is a list** and *NIL* otherwise.

char?

number?

Binary

<, >, = Arithmetic Less than, greater, =, (> x y)

eqv? Non Arithmetic

Conditionals

cond is the conditional operator in Scheme. Its syntax is:

(cond (c_1 f_1) (c_2 f_2) ... (c_k f_k))

The c_i are S-expressions for conditonals *(ex. (> a b)*. which evaluate to true #t or *false #f (often NIL or ()),*

The f_i are *S*- expressions which return values ex. functions or constants .

First c_1 is evaluated. If it is true then f_1 is evaluated and **returned** as the value of the *cond*. Otherwise, c_2 is evaluated and checked for truth value etc. We continue to evaluate c_3, c_4, c_5 ,... until we find the first true value. Then the corresponding expression f_i is evaluated and its value returned as the value of the *cond*. Often c_k is ***else***, which always evaluates to true so that the last case "catches" all other cases. If no c_i evaluates to true, then the cond expression returns *NIL*.

(if (C1) (E1) (E2))

If condition C1 is satisfied do E1 else do E2

Arithmetic

+, -, *, /

© 2006 M C Paull

Definition

(define x '(+ 1 x)) defines a list

(define (add1 x) (+ 1 x)) defines a function This function acn be used; for example, *(add1 2)* would **return** *3*.
We can define a function where its call is needed without actually naming using the anonymous lambda.
(define (third l) (car (lambda (z) (cddr z) L))) *z is a formal argument of function, cddr is applied to that argument and this function is applied toLl.* Here the lambda (**λ**) expression* defines a function which finds the sublist of the original list headed by the third element and continuing thriugh the rest of the list.

Comments*; (semicolon)* Is the *comment* character. If it appears anywhere on a line, the characters to the right of it are ignored as they are a comment.

Eliminating Common Subexpressions Nested Calls and common subexpressions

(let (x_1 E_1) (x_2 E_2) ... (x_k E_k) F)

First evaluate each ***Ej*** independently as though the others did not exist <u>***F*** which is an S expression involving</u> ***x_k s*** is then evaluated with the value of ***E_k replacing x_k***

The value of ***F*** is returned by the ***let*** S expression

(let* (x_1 E_1) (x_2 E_2) ... (x_k E_k) F)

Evaluate each E_j in order **valuing any earlier computed x_{j-k} appearing therein with that of E_{j-k}** and finally ***F*** valuiingeach ***x_k*** appearing therein with that of ***E_k***. The value of *F* is returned by the ***let**** expression

<u>First Class Functions:</u>*<u>Built-ins: map (map f L) applies f to every element of L).</u>*
<u>(remove_if cond L) removes every element in L that satisfies cond</u>

```
(define (sq x) (* x x))
( define (y '(1 2 3)) )
(map   sq  y )
```
Or with a lambda definition**
```
(map  ( lambda (x) (*x x))  y )
```

The function (map f x) can be defined by the user because an actual parameter can be a function. Its definition for any funtion f is:

```
(define  (map   f   x)
         (cond ((null? x) nill)
         ( else (cons ( f(car x)) (map f (cdr x))))
```

```
(define (eq0?  x) (equal? 0 x) )
( define (y '(1 2 0  3)) )
(remove_if    eq0?    y )
```
Or with a lambda definition**
```
(remove_if ( lambda (x) (equal? 0 x)) y )
```

```
( define  (remove_if  fc  x)
   ( cond ((null? x)       nill)
          ( (fc  (car x))  (remove_if fc (cdr x))  )
             (else       (cons (car x) (remove_if  fc (cdr x)) )
```
Use of let:
```
( define  (remove_if  fc  x)
          ( let (  (L (cdr x))  (a (car x))    ;L=cdr x  a = car x
           (    (cond  ( (null? x)    nil )                    )
             (     ( (fc     a)    (remove_if  fc  L )            )
                       ( else        (cons a  (remove_if fc L) ) ) )))
```
The formal parameter fc is a unary condition like equal? or null? in a call of remove_if fc would be replaced by an actual parameter

The special forms ***define* and ***lambda*** allow you to define Scheme functions, named and unnamed, respectively. For example, we can define the function that increments integer values by 1:

Input

read is a function that reads an S-expression from the input and returns that S-expression as its value.

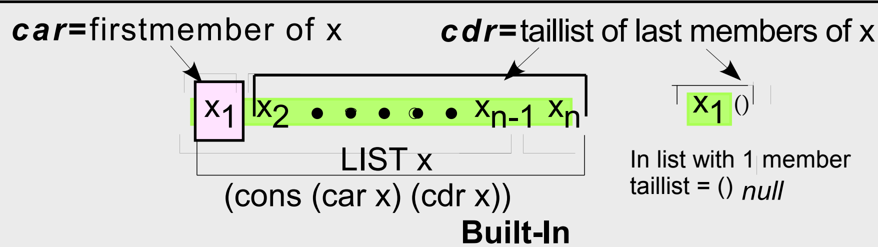
Output

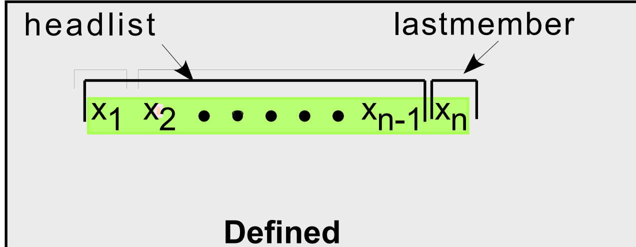
printf is a function that facilitates output of S-expressions. *printf* tis a formatting statement taking a variable number of arguments. The first argument is a control string, delineated by enclosing in double quotes (i.e.,"...") where a ~ in the string indicate as directive or formatting a corresponding element of the arguments in a specific manner. An embedded ~a stands for the value of a variable a; ~% stands for a newline. You can have *printf* control strings with no ~ substrings for printing out messages with no arguments. The sequence of other arguments are values which are printed at the positions set in the control string.

** From the "**λ** calculus" was originally used in demonstrating the universatility of pure functional language by F. Church (@ Princeton). Lisp was based on this notation.

© 2006 M C Paull

Defining Functions In Scheme:
The List is the basic data-structure and there are three fundamental built-in operations which operate on a list.





The (define (name args) .....) is used to define new functions. So we can simply rename built-ins.

{define** (firstmember x) (**car** x) **}   [*firstmember* is the first element of a string]
{define** (taillist x) (**cdr** x) **}   [*taillist* = list after first member is removed]

Defining Functions

(a) [lastmember of list x is the firstmember of x if the taillist (cdr) of x in null (empty)
otherwise it is the lastmember of taillist of x]

(b) lastmember(X) = firstmember(X)   :if taillist(X) = ().
lastmember(X) = lastmember(taillist(X))   :if taillist(X) != ().

```
(c) 1 {define ( lastmember x) (cond ( (null? (taillist x)) (firstmember x) ) *
                          (else          (lastmember (taillist x))  ) ) } Tail Recursion
   1'  {define( lastmember x )
             ( cond  ( (null? (taillist x)) (firstmember x) )              Use Of Let
                   (else   (let ( (y (taillist x)) ) (lastmember y) ) ))}
```

EXAMPLE CALL
Tail Recursion

	Because
1) (lastmember '(a,b,c))=	taillist (a,b,c)) = (b,c)
(lastmember '(b,c))=	(taillist (b,c)) = (c)
(lastmember '(,c))=	(firstmember c) =c

Example Of Definition (lastmember): Three representatives (a) English, (b) Mathematical (c) Scheme

(a) [headlist of list x = x if x has 1 member(taillist =())
other wise = list of(firsmember of x || headlist list x- 1st member of x)

(b) headlist(X) = X)   :if taillist(X) = ().
headlist(X) = cons (fistmember(X), headlist(taillist(X)))   :if taillist(X) != ().

```
(c) 2  { define (headlist x) (cond (( null? (taillist x) ) ()) *
                          (else         (cons (firstmember x) (headlist (taillist x))))}
   2'  {define (headlist x)
            (let  ( ( y  (taillist x) )  (z (firstmember x) )   )
                 (cond  ( ( null? y )  () )                       Use Of Let
                      (else    (cons z  (headlist x)  )  )  ) }
```

EXAMPLES CALL

	Because
2) (headlist '(a,b,c)) =	
(cons a (headlist (b,c)))=	(taillist (a,b,c)) = (b,c)
(cons a (cons b (headlist (c))))=	(taillist (b,c)) = (c)
(cons a (cons b ())) =	(headlist (c)) = ()
(cons a (b))) =	(cons b ()) = (b)
(a b)	(cons a (b))) = (a b

Example Of Definition (heaqdlist): Three representatives (a) English, (b) Mathematical (c) Scheme

© 2006 M C Paull

There is a built-in append function, but this may also be defined using the basic list functions car = firstmember, cdr = taillist and cons.

```
(define (append X Y ) (cond ( (null? X)                Y                   )
                             ( else   (cons (firstmember X) (append (taillistl X) Y) ) )      T
                      )
)
```

```
                                                          PROLOG PARAPHRASE
append ( X, Y, Y ) :-  null?( X)                          )
append ( X, Y, Z )  :- firstmember( X,A) , taillistl X,B) ,append (B,Y, C), cons(A,C,Z).
```

```
(define (revs L) (cond ( (null? (taillist L))              L                  )
                        (     else          (append (revs (tail L)) (list (first L))) )      NT
                 )
)
```

$(rev\ x_1\ x_2 \ldots x_{n-1}\ x_n) = (rev\ x_2 \ldots x_{n-1}\ x_n) \,||\, (x_1)$

```
(revs  (a b c )) =  (append  (revs ( b c)) (list                 a) )
                    (append  (revs  (b c))                      (a) )
                    (append  (append  (revs (c) ) (list b) )  (a) )
                    (append  (append  (revs (c) )  ( b)    )  (a) )
                    (append  (append  (c) )  ( b)    )  (a) )
                    (   c   b    a )
```

```
(define (revs L) (cond ( (null? (taillist L))              L                  )
                        (     else          (append ((list(lastmember L)) ( revs (headlist  L))      T
                 )
)
```

$(rev\ x_1\ x_2 \ldots x_{n-1}\ x_n) = (x_n) \,||\, (rev\ x_2 \ldots x_{n-1})$

```
(revs  (a b c )) =  (append  ((c) (revs (a b)                  ) )    list lastmember steps omitted
                    (append  ((c) (append  (b)  (revs (a) ) ) )
                    (append  ((c) (append  (b)  (    a      ) ) )
                    (c b a)
```

Conditionals: Functions which return #T (True) or #F (false)

```
(define  (size? L p) (equal? (length L) p)      )
(define  (size L) (cond ( (size? L 0)  0                        )
                         (     else      (+  (size (cdr L))    1))
                  )
(define (member? x L) (cond ((null? L)  #F)
                            ((equal? x (car L)) #T)
                            (  else    (member? x (cdr L)))
                      )
)
(define (rp? L) (cond ((null? (cdr L))            #F   )         [Look for repeats in list]
                      ((member? (car L) (cdr L))       # T   )
                      (       else                 (rp? (cdr L)))
                )
)
```

More complex functions can be developed top-down. The final fuction F is defined assuming the existence of simpler functions Then those simpler functions are defined. This is done in a sequence of stages. In the sorting examples that follow the final sorting function defined in terms of simpler functions is the last one defined-the simpler funv=ctions are designed earlier in stages with the simpler functions designed earliest.

A. To Build a Sorter: By finding largest member of unsorted list and putting in front of sorted version
1. Find Largest member of list L (MAX) maxlst(L) returns MAX
2. Remove a member(MAX) from list rmv(X,L) returns L - MAX
3. sort(L) = MAX || L - MAX

returns (+ a1 (+(a2 (Lsum a3))]

```
(define (bigger X Y) (cond ( ( > X Y)    X  )                [ > built-in (type by operator)]
                           ( else       Y  )
        )

(define (maxlst L) (cond ( (null? (tail L))        (first L)          ) [ returns single element in list]
                         ( else          (bigger (first L) (maxlst (tail L))) )
        )
)
(define (rmv X L) (cond ( (null? L)                ()                  )    [ if L empty returns()]
                        ( (= (first L) X)        (tail L)              )    [if L has 1 member]
                        ( else           (cons (first L) (rmv X (tail L)) ) )   [returns a list with (one)
        )                                                                    maximum   removed]
)
(define (sort L) (cond ( ( null? (tailistl L) )            L                  )
                       (  else        (cons (maxlst L) (sort (rmv (maxlst L) L))) )      max first
        )
)
```

Bottom

Top

Use (remove_if ?null L)

```
(define (sort1 L) (cond ( (null? (taillist L))                              L                  )
                        ( else  (let ( (max  (maxlst L) )   (cons max ( sort1 (rmv max L) ) ) )  )
                        )
)
```

Use Of Let

```
( sort                        [5 3  4]                                  )
(cons (maxlst [5 3 4 ]) (sort (rmv (maxlst [5 3 4]) [5 3 4] )  )  )
(cons        5          (sort (rmv        5        [5 3 4] )  )  )
(cons        5          (sort                [3 4]                    )  )
(cons        5          (cons (maxlst [3 4]) (sort (rmv (maxlst [3 4]) [3 4] )  )  )
(cons        5          (cons      4         (sort (rmv       4        [3 4] )  )  )
(cons        5          (cons      4         (sort           [ 3]              )  )  )
(cons        5          (cons      4         (cons (maxlst [3]) (sort (rmv (maxlst [3]) [3] ) )  )
(cons        5          (cons      4         (cons      [3]     (sort (rmv    [3]       [3] ) )  )
(cons        5          (cons      4         (cons      [3]     (sort      []    )  )  )
(cons        5          (cons      4         (cons      [3]           []         )  )  )
(cons        5          (cons      4                    [3]                      )  )
(cons        5                           [4 3]                                   )
                        [5 4 3]
```

© 2006 M C Paull

B. To Build Sorter: By Merging 2 already sorted lists
1Given 2 already sorted lists, L1 and L2, comparing pairs of members putting greatest in front of result
2. Find first and 2nd half of a list (firsthalf L), lastthalf L), returns halfs
3. sort(L) = sortmerge two halfs, and merge(firsthalf L)

(merge L1 L2): if either L1 or L2 is null return other if first of L1 is bigger than first of L2 return first of L1 cons-ed with merge of tail(L1) and L2

```
(define (merge L1  L2)  (cond  (         (null? L1)                        L2                 )
                               (         (null? L2)                        L1                 )
                               ( (> (first L1) (first L2) )   (cons (first L1) (merge (tail L1) L2 ))  )
                               (           else               (cons (first L2) (merge L1 (tail L2) ))  )
                        )
)
```

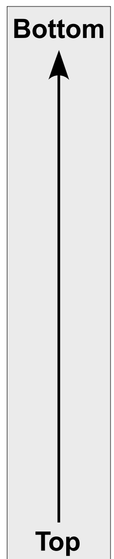
Trace 1

```
(merge  (3 1 )  (6 5) )
=  else   ( cons ( 6  (merge ( 3 1)  (5) )
          ( cons (6 (cons 5 (merge (3 1 ) () )  ) ) )
          ( cons (6 (cons 5  (3 1 )   ) )  )
          ( cons (6 ( 5 3 1 ) )  )
          (6 5 3 1 )
```

Trace 2

```
(merge  (6 )   (5) )
( cons 6 ( merge () (5) )  )
( cons 6 (5)  )
(6 5)
```

```
(define (dec n)    (- n 1)  )
(define (hlfn L)   (/ (length L) 2)  )
(define (hlfn0 L)  (floor (hlfn L)) )
(define (hlfn1 L) (ceiling (hlfn L)) )

(define (firstof n L) ( cond (  (= n 0)                   ( )                      )
                         (  else     (cons (first L) (firstof  (dec n) (tail L))) ) [returns  list   consisting of
                      )                                                               the first n elements of list L]
)
(define (firsthalf L) ( firstof (hlfn0 L) L ))

(define (lastof n L) ( cond ( (equal? n (length L))          L              )
                         (    else                  (lastof n (tail L) ) )
                     )
)
(define (lasthalf L) ( lastof (hlfn1 L) bL) )

(define (sortmerge L) ( cond ( (null? L)                                  ( )                          )
                           (  (null? (tail L))                            L                            )
                           (      else        (merge (sortmerge (firsthalf L)) (sortmerge (lasthalf L))) )
                      )
)
```



[break list in half sortmerge each half and the merge those sortmege d halfs.]

Trace

```
sortmerge  (3,1,5,6) = (merge (sortmerge  (3,1), sortmerge  (5,6),)
= (merge (merge(sortmerge(3), sortmerge(1) ) (merge(sortmerge(5), sortmerge(6) )
= (merge (merge (3), (1) ) (merge (5), (6) )
= (merge ( (cons 3 (1 ) ) (merge (cons 6 (5) )
= (merge ( (3 1 ) ) (merge ( 6 5) )
```

© 2006 M C Paull

Functions Can Be Arguments:
Functions can be defined which take functions as an argument. Here is a function, applst1 which is used to apply another function, f, to every member of a list to produce a new list

```
(define (applst1 f L) ( cond ( (null? L)                ( )                          )
                              ( else      (cons (f (firstmember L) ) (applst1 O (tailistl L)) ) )
                        )
)
```

Applying f to the ith member of 2 lists to get the ith member of a resultant list

```
(define (applst2 f L1 L2) ( cond ((null? L1)                          ( )                          )
                                ( (null? L2)                            ( )                          )
                                (else    (cons (f  (firsmember  L1)  (firsmembert L2) )) (applst2 f
(tailist l L1) (tailistl L2))))
                                )
)
```

So given the additional two definitions:
(define (double X) (2 X)) and (define (sum X Y) (+ X Y))*
We can define:
(applst1 double L) return a list, L2, in which each entry is twice that of the entry in L
(applst1 sum L1 L2) return a list, L3, in which each entry is the sum of the corresponding members in L1 and L2.

The Builtin *map* Function and lambda system for Local Function Definition
There is, in fact, a built- in function,**map** which can be used to apply a function, f, to a list L.
This is the map function.

(***map*** *f L)* [returns result of appying f to each member of L]
(***map*** *double L)* [multiplies each member of L by 2 using double as defined above]

```
(define (incr1 x) (+ x   1))

(define (incr L) (cond ( (null? L)            '()                          )
               (         (else          cons (incr1 (car L))  (incr (cdr L))) )
         )
)
(define (Lincr L) (map incr1 L)   )
```

There is also a way of defining a function locally:
lambda *(x) (f x)* in place of a definition of a function returns result of applying f to x

USES OF *lamda* and *map*
(define (mapconcat L) (map (lambda (x) (cons x '(a))) L)) [concatenatess a to every member of L]

(define (Ldouble L) (map (lambda(x) (2 x)) L))* [multiplies each member of L by 2 like like above,g but a seperate definition of double is not required]

© 2006 M C Paull

Cumulating Functions

if *L = (a1 a2 a3)* a cumulating function, say *(cum f L).* returns *(f a1 (f a2 a3))]*

```
(define (cum f  L)      ( cond ( (null? (tail  L)  )             (first L) )            )
                               ( else      (f (first L )   (cum f   (tail L))) )
                        )
)
```

Or one can define a cumulating function for a particular function, f, ex. +

T

```
(define (Lsum  L) ( cond ( (null? ( tail L) )                 (first L) )
                         ( else         ( + (first L) (Lsum (tail L)) ) )        [ Lsum if L = ( a1 a2 a3)
                                                                                   returns (+  a1 ( (Lsum a2   a3 ))]
                   )                                                              returns (+  a1 (+( a2  (Lsum  a3 ))]
)                                                                                 returns (+  a1 (+( a2   a3 ))]
```

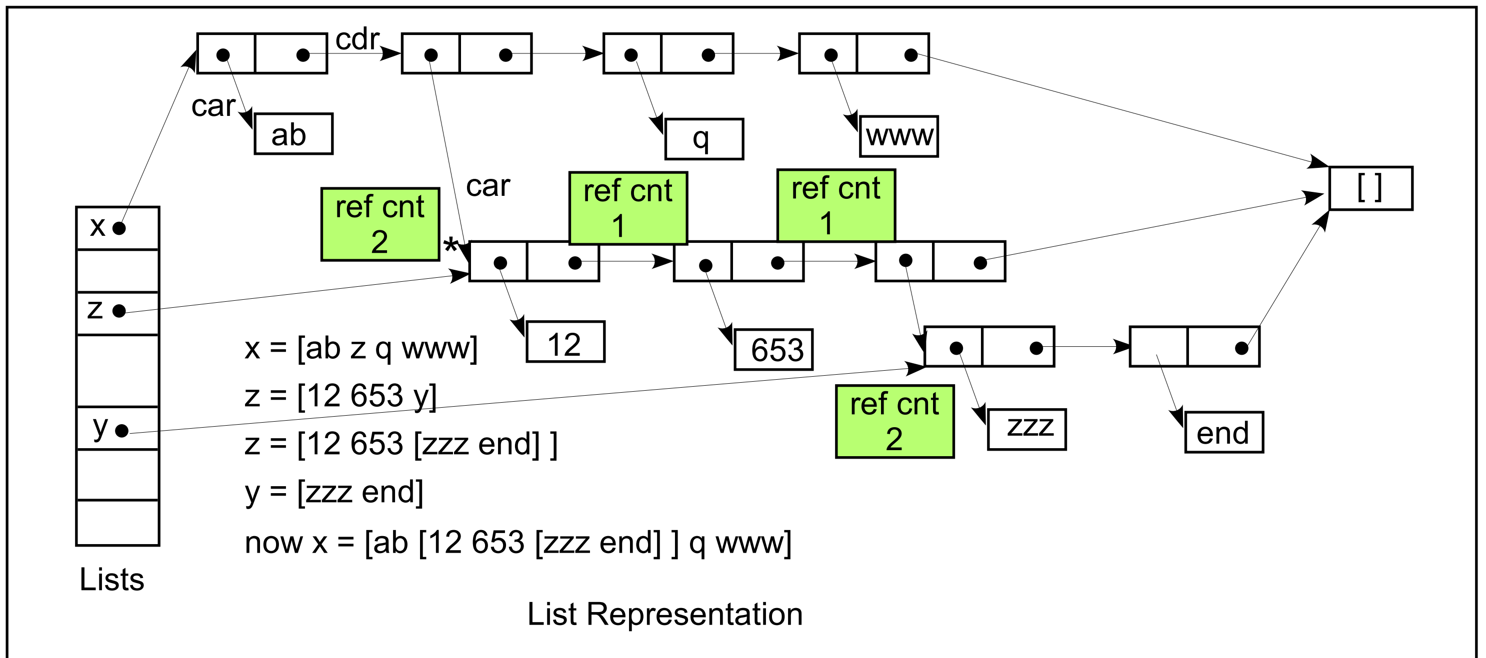
All Permutations of the first n digits

IDEA:

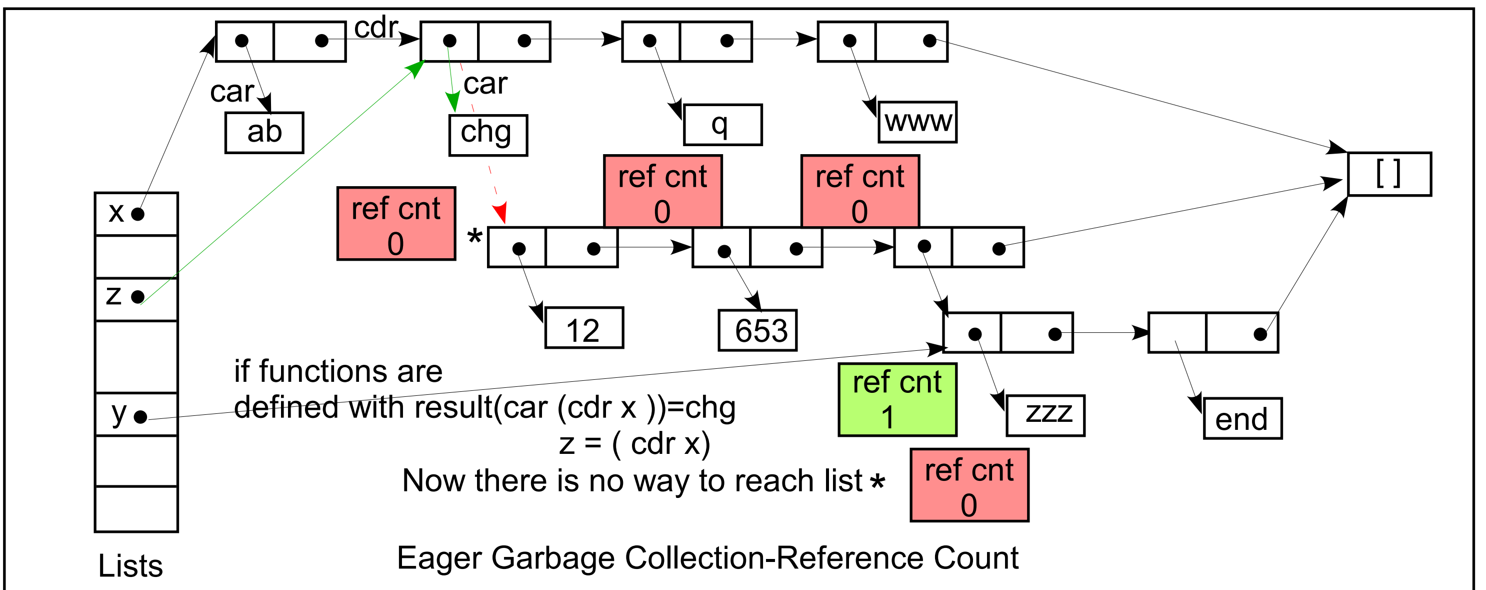
```
(    (1)                                                                                  )
(   (21                                                   (12)                            )
(   (321)          (231)          (213)          (312)          (132)          (123) )
(   (4321) (3421)  (3241)
```

```
(define (first n L) (cond ((= n 0) ‘())
                          (else ( cons (car L) (first (- n 1) (cdr L))))
                    )
)

(define (remain n L) (cond ((= n 0) L)
                           (else (remain (- n 1 ) (cdr L) ))
                     )
)

(define (concat3 L1 n L2) (append L1 (cons n L2)))
(define (insert1 n p L) (concat3 (first p L) n (remain p L)))


(define (insert n p LL) ( cond ((null?  LL) ’())
                               ( else  (append (list (insert1 n p (car LL))   (insert  n p (cdr LL))))
                        )
)

(define (insertAll  n p LL) ( cond ((<  p 0) ‘())
                                   ( else  (append (insert n p LL)   (insertAll n (- p 1) LL)))
                            )
)

(define (inAll n  LL)   (insertAll n (- n 1) LL) )
(define (innest n)      (cond ( (= n 1) ’((1)) )
                              (  else    (inAll n ( innest (- n 1) ) )
                              )
                        )
)
```

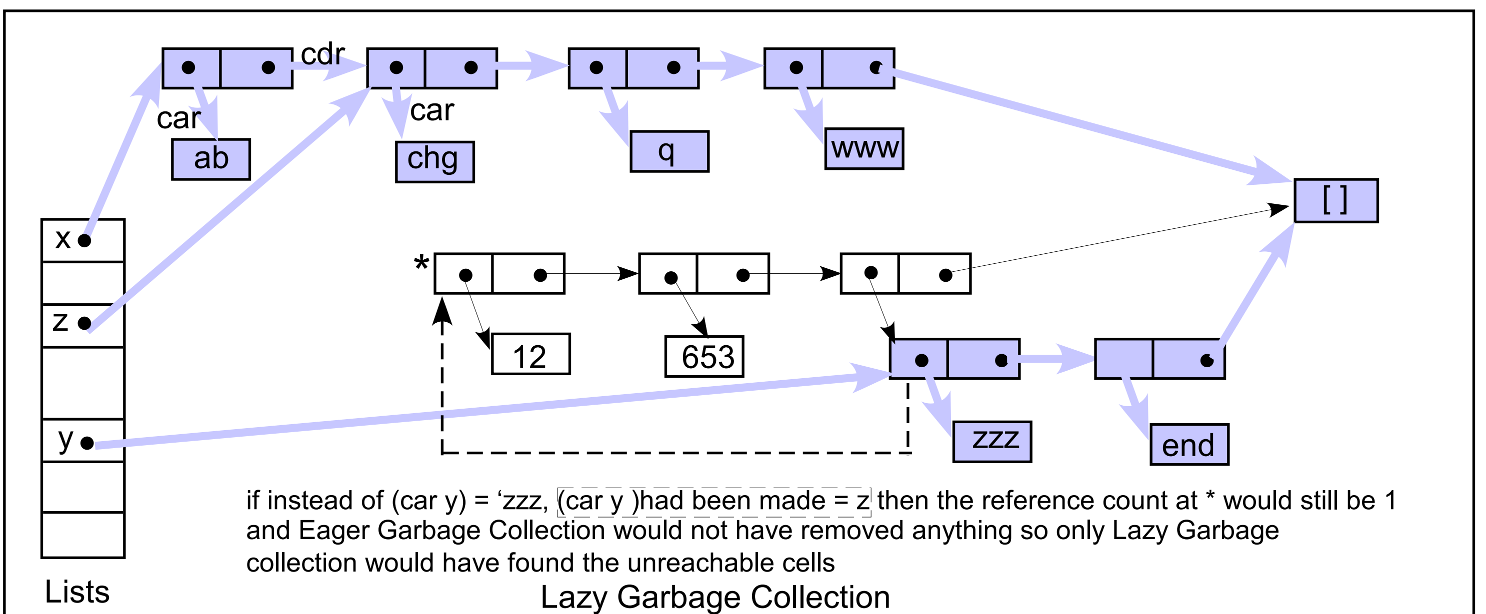
© 2006 M C Paull



List Representation



Eager Garbage Collection-Reference Count



Lazy Garbage Collection

REPRESENTATION OF DATA: LISTS and LIST CONTENTS IN SCHEME

© 2006 M C Paull

Fibonacci Original Recursive Definition Not Tail Recursion

f(n) = 1 : n =0, 1
f(n) = f(n-1) + f(n-2) :n >1

Fibonacci Recursive Definition Tail Recursion

f2(n) = g(0,1,n)
g(a,b,n) = a :n = 0
g(a,b, n) = g(a+b, a, n-1) :n > 0

f2(5) = g(1, 0, 5) = g(1,1, 4) = g(2,1,3) = g(3,2,2) = g(5,3,1) =g(8,5,0) = 8

Because its Tail Recursion It can be implemented by a simple while

```
procedure f2(n)
      {return g(1,0,n); }
procedure g(int a,int b,int n)
while(n > 0)
   { temp = a + b;
     b = a;
     a = temp;
     n=n-1
    }  return a;
```

Greatest Common Divisor Tail Recursion

a,b are integers
1 h(a,b) = a :a = b
2 h(a,b) = h(a-b, b) :a > b

Proof that it gives Greatest Common Divisor.

Claim

h(a,b) = gcd(a, b) :a > b that is : a/h(a,b) = integer & b/h(a,b) = integer and h(a, b) is the largest such divisor)

Proof:

1 is obvious

2 gcd(a, b) =? gcd(a-b, b) :a>b

Need to prove that gcd(a,b) = gcd(a-b,a) if a > b

by definition: a/gcd(a,b) = int1, b/gcd(a,b) = int2 and gcd(a, b) is the largest such number)

therefore the same gcd(a,b)

(a - b) /gcd(a,b) = a /gcd(a,b) (a - b) /gcd(a,b) = int1-int 2 = int3 and since integer 3 divides (a-b) it is = gcd(a-b, b).

3 Like 2

Because its Tail Recursion It can be implemented by a simple while

```
procedure h(int a,int b)
while(a != b)
   { if(a >b) a = a-b;
     if(a< b) b = b-a
    } return a;
```


© 2006 M C Paull