

CS513: Homework 7 Solution (Sketch)

1.
 - (a) An instance can be found in Uri Zwick's paper: The smallest networks on which the Ford-Fulkerson maximum flow procedure may fail to terminate. A draft of this paper is available on his website.
 - (b) Just replace a vertex by an edge with that capacity. You have to take care of incoming and outgoing edges.
 - (c) No. Coming up with a counterexample is easy.
2. Do an induction on d . Base case is trivial. To reduce from a d regular graph to a $d - 1$ regular graph, use Hall's theorem to argue about the existence of a perfect matching and remove that matching.
3. We have to modify the definition of the residual graph to include an extra parameter of the cost of an edge. Now argue that a "negative cost cycle" of flow in the flow network implies that the flow is not of optimal cost. These kind of cycles can be found using the Bellman-Ford algorithm. When we do not have a negative cost cycle, we are done.

Another easy way to solve the problem is by formulating it as an LP.