

Fall 08. CS513. HW 9, Due Nov 25.

1. Given n items such that item i has size s_i , and bins of size 1 each, the problem is to pack the items into the smallest number of bins possible. Formally, *partition* items into sets (bins) B_1, \dots, B_m such that $\sum_{i \in B_j} s_i \leq 1$ and m is as small as possible. Partition means each item is in only one set.
 - Consider the next fit algorithm (NF). It keeps filling bins $1, 2, \dots$, keeping only one bin open at any time (Say bin i is open at some time. All bins B_1, \dots, B_{i-1} are partially filled and closed, and bins B_{i+1}, \dots have not been opened yet). If the next item fits the current open bin i , the item is placed in to it and the bin is left open; else, the bin i is closed and the new bin $i + 1$ is opened for the item. Say NF is the number of bins used by this algorithm on an input and let OPT be the optimal number of bins (smallest possible, for the given input). Show an upper bound on the approximation of NF, ie., $\frac{NF}{OPT}$ in the worst case over all inputs.
 - Show an input on which the algorithm NF achieves its worst case approximation ratio from above.
 - Propose two other algorithms for this problem, without analyzing them. Which do you think of the algorithms has the best approximation ratio? You don't have to prove the claim.
2. Consider the dual set cover problem from HW8.
 - Write an IP to solve the problem.
 - Use LP-based rounding to obtain an approximation. What is the best approximation you can get?
3. Given a set S of axis-parallel rectangles with corners on the integer grid $[1, n] \times [1, n]$, a *stabbing* is a collection of R grid rows and C grid columns such that each rectangle in S has one of these rows or columns pass through its interior. Our goal is to minimize $\max\{|R|, |C|\}$. Design an $O(\log n)$ approximation algorithm for this problem.