Fall 08. CS513. HW 8, Due Nov 20.

1. (Set Cover)

   (a) Input is a collection $\mathcal{S}$ of sets $S_1, \ldots, S_n$. We want to solve the *dual set cover problem* where given an integer $k \leq n$, the problem is to determine a set $\mathcal{T}$ of k sets from the collection $\mathcal{S}$ such that

   $$|\bigcup_{S_i \in \mathcal{T}} S_i|$$

   is maximized. Use the greedy algorithm for the set cover we discussed in the class, and analyze the approximation it gives for this problem.

   (b) Consider the set cover problem with sets $S_1, \ldots, S_n$, and say we have for all i, $|S_i| \leq \alpha$. Then, show that the greedy algorithm discussed in the class is a $(\log_e \alpha) + 1$ approximation.

2. Given a rooted tree T (not necessarily binary), design *two different* algorithms to find the maximum independent set. How much time do the algorithms take?

3. (Coloring)

   (a) Given an undirected graph G, the problem is to determine if it is 3-colorable, that is, is there a way of coloring each vertex using one of the colors R or B or G so that no two vertices of the same color are connected by an edge? Solve the 3-SAT problem by generating an instance in polynomial time of the 3-colorability problem.

   (b) Given a graph G, determine if it is 2-colorable. How much time does your algorithm take?