Fall 08. CS513. HW 7, Due Nov 13.

1. (Simple)

   (a) Can you describe an instance of the maximum flow problem for which the Ford-Fulkerson algorithm may take an infinite number of augmentations?

   (b) Solve the maximum flow problem where in addition to the standard input, each vertex has a capacity of the maximum amount of flow that can go through it.

   (c) If you add a number to each of the edge weights in a graph, do the shortest paths between any pair of vertices remain the same?

2. If every vertex in a bipartite graph has precisely degree d, prove that the graph has d *disjoint* perfect matchings.

   Extra credit. Design an efficient algorithm to find a perfect matching in such a bipartite graph.

   Note. For students interested in theory/combinatorics research, see

   http://www.math.tau.ac.il/ nogaa/PDFS/arr7.pdf

   for fun.

3. Consider the maximum flow problem. There is a non-negative cost function $co(u, v)$ on edges (u, v). The cost of a flow f is

$$\sum_{f(u,v)>0} co(u, v) f(u, v).$$

   Given a real value V, design and analyze an algorithm to find a flow f of minimum cost with value V.

   Hint. Say you have a flow of value V of some cost. Find a suitable *cycle* of flow and decrease the cost. When does such a cycle exist and how do you find it?