

Fall 08. CS513. HW 2, Due Sept 25. You can assume any of the algorithms we discussed in the class: you should precisely state any such result you use without going into details. Unless otherwise mentioned or you can justify the assumption, use comparisons on the array elements for sorting.

1. Simple exercises:

- (a) Given k arrays S_1, S_2, \dots, S_k of possibly varying sizes, give an algorithm to sort each of the arrays. How much time and space does the algorithm take?
 - (b) The *mode* of an array S of numbers is the number that occurs most frequently in S . How much time and space does it take to calculate the mode? Suppose one of the numbers occurs at least $n/2 + 1$ times in S . Now how long does it take to calculate the mode?
 - (c) The problem is to find the k th ranked item (item which will be the k th smallest in the sorted order) in array $A[1, \dots, n]$ of numbers. Design a quicksort style *randomized* algorithm for this problem and analyze its expected running time.
 - (d) You have to sort the array $A[1, \dots, n]$ of integers each in the range $0 \dots m$. You can think of each integer as k -tuple of numbers each in the range $[0, m^{1/k}]$, and sort the k -tuples (how?). What is the best value k to sort these numbers?
2. Given an array $A[1, \dots, n]$ and k intervals where interval I_i is $[l_i, r_i] \in [1, n]$, output for each interval I_i , the median of items $A[l_i], \dots, A[r_i]$. How much time does your algorithm take?
 3. Given an array $A[1, \dots, n]$, design an algorithm to find the largest and the second largest numbers in A . Use as few comparisons as possible.
 4. Given an array S of n numbers, a target number W and a parameter k . Design an algorithm to check if there are k different numbers in S that add upto W . How much time and space does the algorithm take?
 5. **Extra Credit.** Given an array $A[1, \dots, n]$ of numbers in the range $1..n$ where each $A[i]$ is a $\log n$ bit word, sort the numbers *in-place*, that is, using no more than $O(1)$ extra words.