

CS513. Algorithms and Data Structures Practice Exam

(Thursday Dec 11, 2008. 1.45 PM to 3.45 PM, Total Points: 65)

- State your solutions clearly and succinctly. If you have any questions, ask me.
- There are partial credits. Write down the best solution you have, and if you can point out what is your difficulty in getting a better result, you will be given partial credits. Writing "I do NOT know." will get you 1/4th of the points.
- Good Luck.

1. (*Sundries*) [5+5+5+5+5=25 Points]

- (*Divide and Conquer*) A complex algorithm for solving the **bizarre** problem divides the n -sized problem into c subproblems of size $O(n/3)$ and combines the solutions in $O(n^2)$ time. What should c be for the running time of the algorithm to be $o(n^{2.9})$?
- (*Randomized Algorithms*) Say you have n numbers in an array. The goal is to return an item that is in the largest half of the numbers. The algorithm is to pick 10 numbers uniformly randomly and return the largest of these 10 numbers. What is (a lower bound on) the probability that the algorithm succeeds?
- (*Memory*) Pick one problem you liked that was discussed in the class *after* the midterm exam. State the problem and properties of the algorithm we analyzed in the class.
- (*Reduction*) Suppose we found a polynomial time algorithm for the 3-SAT problem. Does it imply a polynomial time algorithm for every problem in NP? Explain your answer formally.
- (*Speculation*) Given a text matrix $T[1..n, 1..n]$ and a pattern matrix $P[1..m, 1..m]$, $n \geq m$, how much time will it take to find all occurrences of pattern in the text? Explain why you believe your answer. I am not looking for an algorithm, just a sentence or two to describe your intuition.

2. (String Matching) [5+15=20 Points] Given a string $s[1 \cdots n]$, the problem is to determine for each i , if it has a substring of length at least 2 that begins at location i and is a palindrome, that is, for each i , find if there is an $i_j \geq 2$ such that $s[i \cdots i_j]$ reads the same forwards and backwards.

- (a) Describe a straightforward algorithm for this problem. Precisely state the algorithm and its running time.
- (b) Present an efficient algorithm for this problem using the Karp-Rabin fingerprint scheme. Precisely state the algorithm, running time and its performance guarantee.

3. (Approximation Algorithms) [20 Points] Given a graph $G = (V, E)$, the task is to assign a color to each vertex from $\{R, B\}$ such that one maximizes the number of edges that have different colors assigned to their endpoint vertices. Derive a randomized approximation algorithm for this problem and analyze its performance.

Hint. The randomized algorithm is to assign each vertex one of the two colors with equal probability. Now what is the expected number of edges that have endpoints of two different colors?