LLE and ISOMAP Analysis of Robot Images

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Abstract

Locally Linear Embedding (LLE) and Isomap are two frameworks to process and analyze high-dimensional non-linear data domains, such as time and spatial series images. These techniques allow the creation of low-dimensional embeddings of the original data that are much easier to visualize and work with than the initial, high-dimensional data. In particular, the dimensionality of such embeddings is similar to that obtained by classical techniques used for linear dimensionality reduction, such as PCA. In this paper we investigate the dimensionality and the geometric properties of the manifold of image sequences from a moving. At the end, we also proposed some future thoughts.

1. Introduction

Many problems in pattern recognition begin with the preprocessing of high-dimensional signals, such as images of face, or spectrograms of speech. Two popular forms of dimensionality reduction are the methods of principal component analysis (PCA) [6] and multidimensional scaling (MDS) [7]. Both PCA and MDS are eigenvector methods designed to model linear variabilities in high dimensional data. In PCA, one computes the linear projections of greatest variance from the top eigenvectors of the data covariance matrix. In classical (or metric) MDS, one computes the low dimensional embedding that best preserves pairwise distances between data points. If these distances correspond to Euclidean distances, the results of metric MDS are equivalent to PCA.

Both methods are simple to implement, and their optimizations do not involve local minima, but they have inherent limitations as linear methods. If one were to use PCA and MDS to perform analysis on such a nonlinear data as shown in Figure 1(b), the result is that illustrated in figure 1(a).

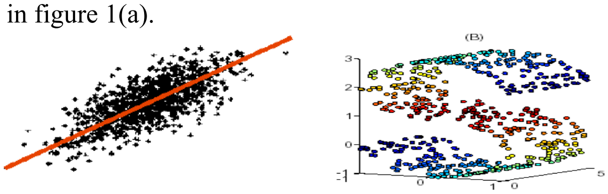

Figure 1. (A) The red line shows the basis vector for the given linear dataset. (B) Example of nonlinear data that PCA and MDS will fail to analysis since it is hard to find out the dominant vectors.

Recently, two nonlinear dimensionality methods Isomap and LLE have been introduced to perform analysis on high dimensional data such as images and spectra. In this paper, we will review these two algorithms and apply them to embed the images got from a Robot dog to a much lower dimensionality.

2. Algorithms

In this section, we are going to introduce the basic concept of Isomap and LLE.

2.1 Isomap

The Isomap (Isometric mapping) technique is based on the idea of viewing the problem of creating a high dimension to low dimension transformation as a graph problem. Isomap can be seen as a way of generalizing Principal Component Analysis (PCA), the classic technique for unsupervised structure discovery in high dimensional spaces. The example of a transformation performed by Isomap is given in Figure 2.

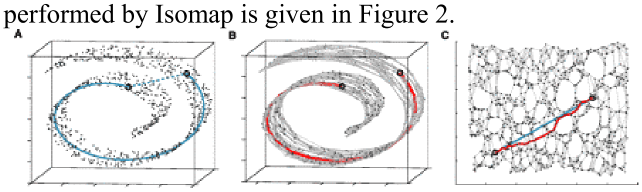

Figure 2 Isomap Technique [4]. The Swiss roll example that demonstrates the transformation and embedding of the 3 dimensional “Swiss Roll” into 2-dimensional space. In image A, for any two points (dark circles) their Euclidean distance (dashed blue line) may not represent their intrinsic similarity, as does geodesic distance (solid blue line). Image B shows two points in the Neighborhood graph G (constructed in Step 1 of the Isomap algorithm) and the geodesic path (red line) between them (constructed in Step 2). Image C shows the embedding (constructed in Step) that best preserves the shortest path distances in the neighborhood graph. The blue line shows the

straight-line distance between the two points and red line shows the geodesic path.

The Isomap algorithm itself consists of three steps:

1) The estimation of the neighborhood graph

Given the input points, find out the neighbors for each point, either by K-nearest neighbors or all those within some radius e. Then a neighborhood graph is constructed, with each edge assigned a weight $d_X(i,j)$, corresponding to a distance between two points.

2) Computing the shortest path graph given the neighborhood graph

The geodesic distances $d_M(i,j)$ between all the connected points in the graph are calculated by computing the shortest path between any two given data points using the Floyd-Warshall algorithm or Dijkstra's algorithm.

3) Construction of lower dimensional embedding

Finally, Multidimensional Scaling (MDS) [5] produces a d-dimensional Euclidean space embedding preserving the original data's intrinsic geometry. The embedding is based on the method of. However, unlike classical MDS, the MDS used for Isomap algorithm is applied to geodesic distances.

2.2 LLE

Similar to Isomap, the LLE technique consists of the three basic parts that correspond to the Isomap algorithm steps.

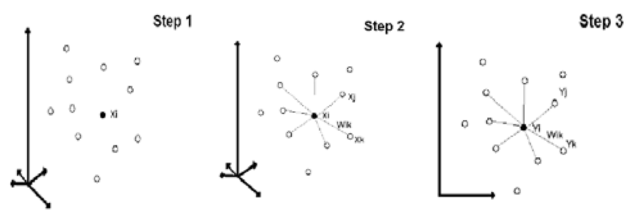

Figure 3. Three steps of the LLE technique. (A) In Step 1 of the algorithm, K nearest neighbors are found for each point. (B) In Step 2, weights Wij are caclulated. These weights describe the position of point Xi in relationship to its neighboring points Xj. (B) In Step 3, embedding in low dimensional space is performed using the weights Wij obtained in the Step2.

The LLE steps are the following:

1) Neighborhood grouping

Find the neighborhood for any given point in the starting space using K nearest neighbors.

2) Calculation of weights

Compute the weights W_{ij} that best reconstruct each data point $\vec{X}_i$ from its neighbors, minimizing the cost in eq. (1) by constrained linear fits

$$\mathcal{E}(W) = \sum_i \left| \vec{X}_i - \sum_j W_{ij} \vec{X}_j \right|^2 \qquad (1)$$

3) Embedding

Compute the vectors $\vec{Y}_i$ est reconstructed by the weights W_{ij}, minimizing the quadratic form in Eq. (2) by its bottom nonzero eigenvectors

$$\Phi(Y) = \sum_i \left| \vec{Y}_i - \sum_j W_{ij} \vec{Y}_j \right|^2 \qquad (2)$$

3. Application

This section presents the use of Isomap and LLE methods in analyzing the 2 datasets of colored images (resolution is 72*88*3) taken by the AIBO® robot dog's digital camera.

3.1 Data Representation and Experiment Results

3.1.1 DATASET 1 (BASELINE SWEEP)

Dataset 1 is a collection of 283 images taken by during a full sweep of its head from right to left without moving. The Isomap and LLE analysis of Dataset 1 are shown in Figures 4, 5 and 6.

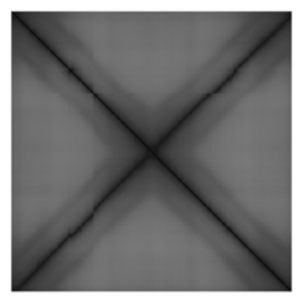

Figure 4. A pairwise Euclidean distance matrix used in Isomap. Note: The pairwise distance is closer to 0 as the grayscale of the point becomes darker. The diagonal dark line shows that the pairwise distance of an images to itself, and the reciprocal diagonal shows there is an exact fold of 283 images, which means No. 1-141 images are the same as No. 283-142.

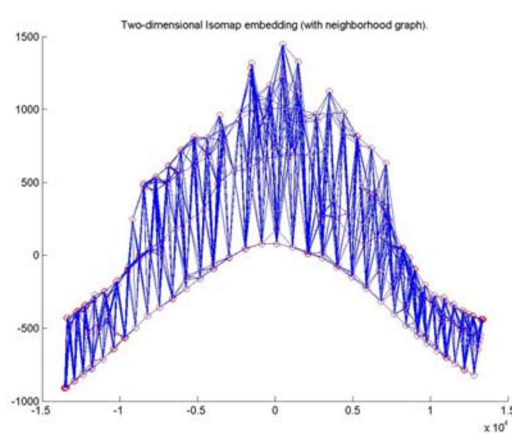

Figure 5. shows the 2-d embedding result of Isomap. It is easy to see the rough shape of embedding is an arch with some outliers beyond it.

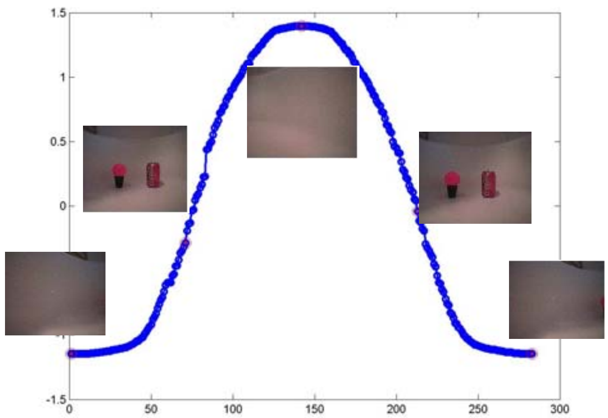

Figure 6. shows the 1-d LLE embedding of 283 images plotting in 2-d style. In this figure, x-coordinate is 283 points, whereas y-coordinate is 1-d embedding value. The result of LLE is better than Isomap since the embedding is very smooth curve, without any outliers.

3.1.2 DATASET 2 (WALKING AND TINY SWEEP)

Dataset 2 is a collection of 849 images taken when the robot is walking and stopped with a tiny sweep. The Isomp and LLE analysis of dataset 2 are shown in Figure 7 and 8.

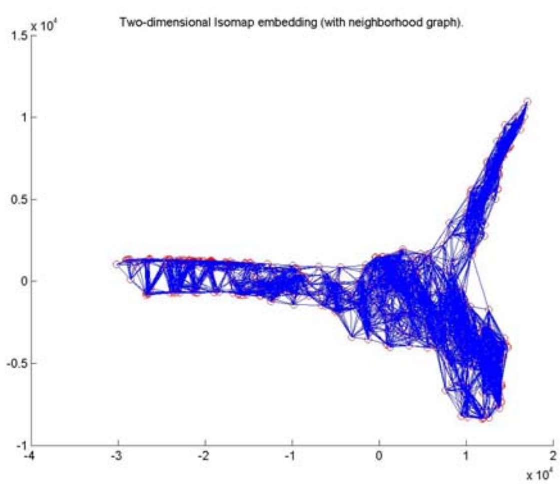

Figure 7 shows the 2-d embedding result of Isomap.

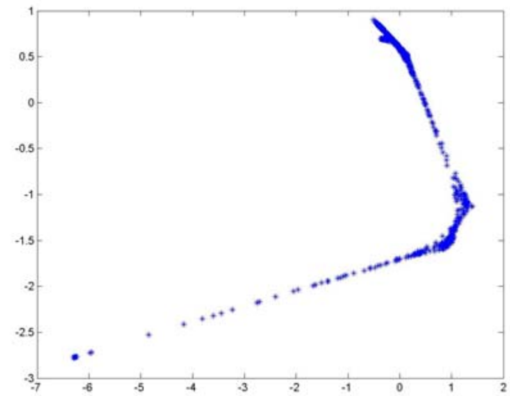

Figure 8. shows the 2-d LLE embedding.

3.2 Isomap vs. LLE

It is shown that Isomap and LLE almost have the same shape of embedding for two datasets. However, LLE has less outlier than Isomap because Isomap sees the dimensionality reduction problem as a global graph problem, in which data are represented as connected graphs and the relationship among data is described through the use of geodesic distances, whereas LLE takes a different approach, preserving locally linear configurations of nearest neighbors. So Isomap can be unstable, depending on the topology of the data, while LLE may have better performance because it is a locally optimal solution. The difference is shown in Figure 9.

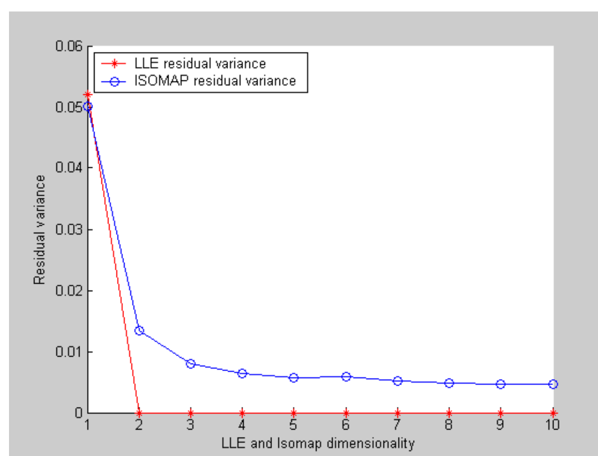

Figure 9 Shows the residual variance of LLE and Isomap by embedding dimensionality for dataset 1. residual variance of LLE drops more rapidly than Isomap, which means training error of LLE is better than Isomap.

However, there is an important advantage of the Isomap algorithm is that it calculates the relationship between the dimensionality and the residual variance. This can be very useful in estimating the intrinsic dimensionality of high dimensional data being processed by looking at the elbow point, which is equivalent to the size of the target embedded space for the data being processed.

4. Summary and Future Thoughts

In this paper, we reviewed two nonlinear dimensionality reduction frameworks (Isomap and LLE), and applied them to the analysis of intrinsic dimensionality of images taken by a robot dog when it is moving. Both methods have advantages and disadvantages. A nice property of Isomap is that it can give an estimation of intrinsic dimensionality of high dimensional data being processed by looking for an elbow point, which is equivalent to the size of target embedded space for the data being processed, but it is very sensitive to outliers, like blury images because of its high dependency on graph topology. As for LLE, it is not capable of estimating the intrinsic dimensionality, but since it is a locally optimal embedding, it is more robust to outliers. That is why LLE embedding appear smoother than those of Isomap.

We have noticed that both Isomap and LLE use Euclidean distance to tell the difference between two images. This is still not robust to the robot dog's images since its walking style is not stable, so it adjacent images blur, or jump back and forth. Even though the images are neighbors, a Euclidean distance calculation will not think of them as neighbors, so that LLE and Isomap can not capture the structure in image space. Our next goal is to find out a more robust distance indicator of similarity in the dataset.

Given such embeddings, next we would like to learn implicit representation of the manifold as well as learning a nonlinear mapping between the embedding space and the visual input space. In the future, based on the mapping, it will be easy to reconstruct the scenario of where the robots dog was, and use the Reinforcement Learning on embedding space to train the Robot where to move next when it sees familiar scenes.

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