A New Evolutionary Algorithm for Multi-objective Optimization Problems

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Abstract

Among the currently successful Evolutionary Multi-Objective Algorithms (MOEAs), elitism and no sharing factor are two common characteristics and have been demonstrated to improve performance significantly. Based on these two principles, two heuristics, with which impressive improvements were showed in single objective optimization, are introduced in a newly designed EMOA in this paper: multi-parent crossover, which ensures that the population converges to the true Pareto optimal front; and swarm hill climbing, which effectively helps prevent premature convergence and achieve a well distributed trade-off front.

1. Introduction

Since the 1980s, evolutionary algorithms have been applied to various multi-objective optimization problems for finding Pareto optimal solutions and have achieved great success. In recent research, a new operator, elitism, was proposed and has been demonstrated to improve significantly the performance of multi-objective algorithms. It maintains individuals with the best fitness in the population or in an auxiliary population. The sharing function approach is helpful to retain diversity of the population, but it needs an appropriately set fitness sharing factor right in order to define the niche size of an optimum.

Based on the above two considerations (elitism and no sharing factor), two heuristics, with which impressive improvements were showed in Single Objective Optimization (SOO), are introduced in a newly designed Evolutionary Multi-Objective Algorithm (EMOA) in this paper: multi-parent crossover, which ensures that the population converges to the true pareto optimal front; and swarm hill climbing, which effectively helps prevent premature convergence and achieve a well distributed trade-off front.

The paper is structured as follows: Section 2 introduces elitism and diversity and how they are implemented in the new proposed MOEA. In Section 3, the new MOEA is presented, with the discussion of the two newly introduced heuristics, multi-parent crossover and swarm hill climbing. In Section 4, computational experiments, followed by discussion and analysis, are described based on three well known test functions.

2. Elitism and Diversity in MOEA

2.1. Elitism

Elitism in EA means that elite individuals, which are not worse than any other solutions in the current population, should not be eliminated during evolutionary process. Elitism has been demonstrated to improve significantly the performance of MOEAs, as can be seen, in SPEA (Zitzler,1999) and NSGA-II (Deb,2000). It guarantees that the fitness of the best solution in each generation should not deteriorate. A good solution found in early generation shall not be replaced unless a better one is generated.

The new MOEA proposed in this paper is a sort of stead-state EA, in which elitism can be introduced in a simple manner. In each run of the evolutionary process, one new offspring is generated by means of a multi-parent non-convex crossover operator over m randomly picked parents. The newly generated offspring is compared with all individuals in the current population to compete for a slot in the next generation, allowing the solutions of the best front to be retained and carried over to the next generation.

2.2. Diversity by Crowding Model

In MOEA, one key issue is how to keep diversity in the population through the whole pareto front. A fitness sharing procedure was employed in early MOEAs, such as NSGA (Srinivas 1994), NPGA (Horn 1994), for this goal. But, the performance of this approach depends on correctly setting the parameter σ_{share}, the fitness sharing factor.

In this paper, a crowding model is employed to keep diversity. Just as the name suggests, in a crowding model, crowding of solutions anywhere in the search space is discouraged, thereby providing the diversity needed to maintain multiple optimal solutions. As for specific implementation for this model, a similar routine as Deb (2001) used in NSGA-II is employed.

3. A New MOEA

Introducing two heuristics, multi-parent crossover and swarm hill climbing, I propose a new elitist steady state approach for multi-objective optimization: each generation, only one offspring is generated by means of multi-parent crossover. One of those individuals dominated by this new offspring, if any, is randomly chosen to be replaced. If no individual is dominated, then the algorithm replaces the lowest rank and the least widely spread solution by using the crowding distance values; the evolutionary process terminates only when the ranks of all individuals are 0 and the difference between the biggest crowding distance value and the smallest crowding distance value is less than some small number ϵ (swarm hill climbing).

3.1. The Flow of the New MOEA

The flow of the new proposed MOEA is outlined in the following.

Step1 A population $P(0)$ is generated at random and $t = 0$ is set;
Step2 Select at random m individuals from $P(t)$ and create a single offspring y by multi-parent crossover;
Step3 Collect all individuals D_y from $P(t)$ that are dominated by y and calculate $y's$ rank and crowding distance value at the same time;
Step4 If $D_y \neq \phi$, delete one of the members of D_y at random and append y in $P(t)$ and go to Step5. Otherwise, append y and delete the lowest rank and the least widely spread solution.
Step5 If the ranks of all individuals are 0 and the difference between the biggest crowding distance value and the smallest crowding distance value is less than some small number ϵ, or the maximum evolution generation is reached, then stop and declare $P(t+1)$ as the set of obtained non-dominated solutions; otherwise set $t = t + 1$ and go to Step 2.

3.2. Pareto Ranking and Crowding Distance

Each solution at generation t has its corresponding objective vector X_u^t. Denote as R_u^t the number of objective vectors in the t generation that dominate X_u^t. Then, $X's$ Pareto Rank is defined by: $rank(X, t) = R_u^t$

The multiple objective values are consequently converted to a ranked metric by using the non-dominating principle. To further evaluate the preference for solutions that share the same rank, a crowding distance will be assigned by using the method proposed by Deb (2000) as follows.

Crowding distance assignment procedure:

Step1 Group the population by the individuals' rank;
Step2 For each member in the same rank group ψ, first assign $d_i = 0, i = 1, 2, ..., k$ $k = |\psi|$;
Step3 For each objective function $m = 1, 2, ..., M$, sort ψ in worse order of f_m or, find the sorted indices vector:I^m
Step4 For $m = 1, 2, ..., M$, assign a large distance to the boundary solutions, or $d_{I_1^m} = d_{I_k^m} = \infty$, and for all other solutions $j = 2$ to $(k - 1)$, assign:

$$d_{I_j^m} = d_{I_j^m} + \frac{f_m^{(I_{j+1}^m)} - f_m^{(I_{j-1}^m)}}{f_m^{max} - f_m^{min}}.$$

The index I_j denotes the solution index of the $j - th$ member in the sorted group ψ. The second term on the right side of the above equation is the difference in objective function values between two neighboring solutions on either side of solution I_j.

Please note that only when a tie appears between two solutions' ranks shall we compare their crowding distance values when evaluating their fitness. And when applying the swarm hill climbing heuristic as a stopping condition, which will be discussed shortly, the crowding distance values of the boundary solutions shall not be considered.

3.3. Multi-parent Non-convex Crossover

Guo (1999) discussed the advantages of multi-crossover. His computational experiments in SOO show that, given a fixed size population, the more individuals get involved in crossover, the greater the probability of convergence to the optimum. This characteristic is also demonstrated by the computational results when I apply it in the new MOEA.

Let $x_i, i = 1, 2, ..., m$ denote m points in space R^d, define a non-convex subspace Vover these m points as following:

$$V = \{X | X = \Sigma_{i=1}^m a_i x_i\}$$

where

$$\Sigma_{i=1}^m a_i = 1 \; and \; -0.5 \leq a_i \leq 1.5, i = 1, 2, ..., m.$$

Multi-cross over is realized by picking up a point from the V space over the m randomly selected solutions from the current population. Note that the coefficients, random variables a_i range from -0.5 to 1.5, which implies that V is a non-convex space based on $\{x_1, x_2, ..., x_m\}$. When iteration times increase ∞, V could cover the whole R^d. So, even though a mutation operator is not employed in the algorithm, this non-convex crossover could still guarantee that global optimum shall not be missed given enough iterations.

3.4. Swarm Hill Climbing

The idea of swarm hill climbing heuristic was proposed by Guo (1999) in SOO. It is realized by setting the stopping criterion of evolutionary process to be when the fitness difference between the best solution and the worst solution is small enough. By this means, more chances are given for individuals that are trapped at the local peaks, to escape. All individuals climb the hills in parallel. Even when trapped at some local peak, they could adapt themselves and communicate with each other by multi-parent non-convex crossover to escape.

In this new MOEA, I am trying to take advantage of this heuristic. The stopping criterion is set as when the difference between the biggest crowding distance value and the smallest crowding distance value of the non-dominated individuals is small enough. Note that the crowding distances of boundary individuals are not compared since they are set to be some big constant. By this specification, we know that the evolutionary process is continuing when the non-dominated individuals are not yet well distributed. Thereby both avoidance of premature convergence and a well distributed trade-off front are obtained at the same time.

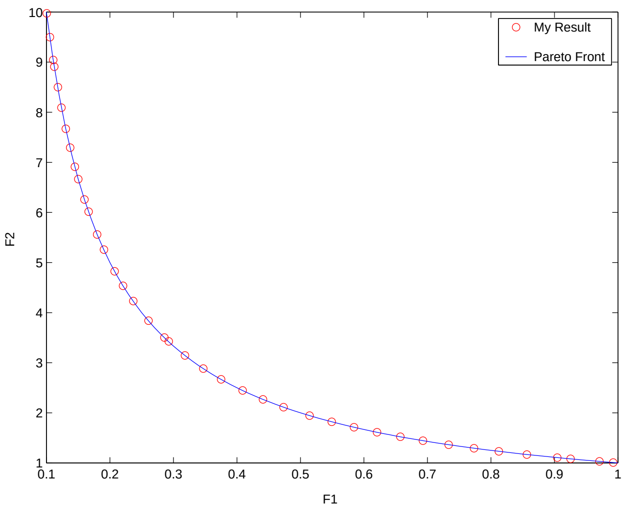

Figure 1. Test Function 1

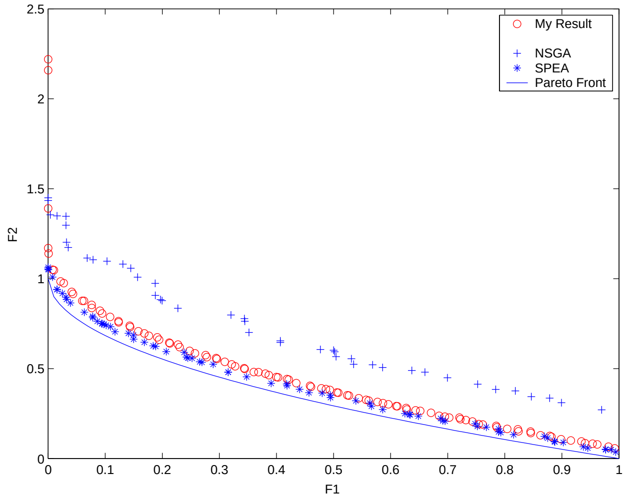

Figure 2. Test Function 2

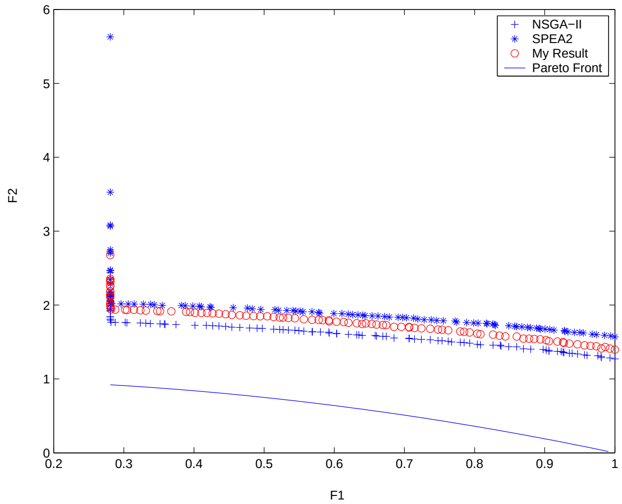

Figure 3. Test Function 3

4. Experiments and Discussion

4.1. Test Functions

Function F_1 is suggested by Deb (2000), who used it to evaluate popular MOEAs. For all pareto optimal solutions, two objective functions f_1,f_2, are related as $f_2 = \frac{1}{f_1}$, $0.1 \le f_1 \le 1$, thereby resulting in trade-offs among the pareto optimal solutions.

Minimize $f_1(x) = x_1$

Minimize $f_2(x) = \frac{1+x_2}{x_1}$

Subject to $0.1 \le x_1 \le 1$, $0 \le x_2 \le 5$.

Functions F_2 and F_3, proposed by Zitzler (1999) are structured in the same manner over three functions f_1, g, h, as follows:

Minimize $F(X) = (f_1(X), f_2(X))$

Subject to

$f_2(X) = g(x_2, ..., x_m)h(f_1(X), g(x_2, ..., x_m))$

Where $X = (x_1, ..., x_m)$.

Function F_2:

$f_1(X) = x_1$

$g(x_2, ..., x_m) = 1 + 9 * \Sigma_{i=2}^{m} \frac{x_i}{m-1}$

$h(f_1, g) = 1 - \sqrt{\frac{f_1}{g}}$,

where $m = 30$, and $x_i \in [0, 1]$. The pareto optimal front is formed with $g(x) = 1$. Function F_3:

$f_1(X) = 1 - exp(-4x_1)sin^6(6\pi x_1)$

$g(x_2, ..., x_m) = 1 + 9 * (\frac{\Sigma_{i=2}^{m} x_i}{m-1})^{0.25}$

$h(f_1, g) = 1 - (\frac{f_1}{g})^2$

Where $m = 10$, and $x_i \in [0, 1]$. The pareto optimal front is formed with $g(x) = 1$ and is nonconvex.

4.2. Simulation results

To make the computational results comparable with other MOEAs, similar values of the parameters as in other MOEAs are used as shown in table 1, referencing Deb (2000), and Zitzler (1999, 2001). As for the number of evolving generation, the steady state evolutionary algorithm differs from other MOEAs since only one offspring is generated each iteration. It is more reasonable to compare runs based on the number of newly generated offspring. The new MOEA has two specific parameters m, the number of parents involved in multi-parent crossover, and ϵ, the threshold for the stopping criterion guided by swarm hill climbing.

Table 1. Parameters used for F1, F2, F3

Fun.	Pop. Size	# Generation	m	ϵ
F1	40	20,000	9	0.01
F2	100	25,000	15	0.01
F3	100	25,000	15	0.01

From the computational result for function F_1 in Figure 1, we could find that resulting solutions are largely distributed along the pareto front. And, at the same time, it indicates a promising potential of good distribution of the solutions. When compared with two early classic MOEAs, NSGA and SPEA, through Function 2, the performance of the new algorithm is much better than the former and comparable with the latter. Its better-distributed-solutions feature is also more obvious. Function 3 has been proposed to test the algorithms' ability to find a good distribution of points in a non-uniformly distributed objective space (Zilter 2001). In this case, again, the new MOEA outperforms NSGA-II and SPEA2, two enhanced versions of NSGA and SPEA, as far as the solution distribution is concerned. Figure 3 also indicates the comparable efficiency between the new algorithm and these other MOEAs.

From the above analysis we might see that the performance of the new algorithm is good and comparable with the results in recent research; furthermore, this new MOEA has its unique advantage: to find well distributed solutions.

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