

Sample Answers to Chapter 5

5.2

(1) Standard Deviation

$$\begin{aligned}\sigma_{\text{errors}(h)} &\approx \sqrt{\frac{\text{error}_s(h)(1 - \text{error}_s(h))}{n}} \\ &= \sqrt{\frac{(r/n) * (1 - r/n)}{n}} \\ &= \sqrt{\frac{((100 - 83)/100) * (1 - (100 - 83)/100)}{100}} \\ &= 0.03756\end{aligned}$$

(2) Confidence Interval

$$\mu = \text{error}_s(h) = \frac{r}{n} = \frac{100 - 83}{100} = 0.17$$

When confidence interval = 95%, the constant $z_N = 1.96$.

Therefore, the confidence interval is $(\mu - z_N \sigma_{\text{errors}(h)}, \mu + z_N \sigma_{\text{errors}(h)})$, which is (0.0964, 0.2436).

5.3

$$\mu = \text{error}_s(h) = \frac{r}{n} = \frac{10}{65} = 0.1538$$

$$\begin{aligned}\sigma_{\text{errors}(h)} &\approx \sqrt{\frac{\text{error}_s(h)(1 - \text{error}_s(h))}{n}} \\ &= \sqrt{\frac{(r/n) * (1 - r/n)}{n}} \\ &= \sqrt{\frac{(10/65) * (1 - 10/65)}{65}} \\ &= 0.04475\end{aligned}$$

Two-sided bounds:

When confidence interval = 90%, the constant $z_N = 1.64$.

90% confidence interval = $(\mu - z_N \sigma_{errors(h)}, \mu + z_N \sigma_{errors(h)})$, which is (0.08041, 0.22719).

One-sided bounds:

Because $100(1-\alpha)\%$ confidence interval with lower bound L and upper bound U implies a $100(1-\alpha/2)\%$ confidence interval with upper bound U and no lower bound. Therefore, 95% one-sided confidence interval and 90% two-sided interval share the same upper bound U.

For 95% one-sided confidence interval, $U = 0.22719$.

Moreover, 90% one-sided interval and 80% two-sided interval share the same upper bound U. For 80% two-sided confidence interval, the constant $z_N = 1.28$. Thus the upper bound U of 90% one-sided interval is $\mu + z_N \sigma_{errors(h)} = 0.21108$.

5.4

Given 95% confidence interval, $z_N = 1.96$;

The range between 0.2 and 0.6 implies that

$$\mu = \frac{0.2 + 0.6}{2} = 0.4.$$

Moreover, $U = \mu + z_N \sigma_{errors(h)} = \mu + 1.96 * \sigma_{errors(h)}$ (1)

$$L = \mu - z_N \sigma_{errors(h)} = \mu - 1.96 * \sigma_{errors(h)}$$
 (2)

(1)-(2) represents the width of the two-sided 95% confidence interval, which is required to be smaller than 0.1,

$$\rightarrow 2 * 1.96 * \sigma_{errors(h)} \leq 0.1$$

$$\rightarrow 2 * 1.96 * \sqrt{\frac{errors(h)(1-errors(h))}{n}} \leq 0.1$$

$$\rightarrow 2 * 1.96 * \sqrt{\frac{\mu(1-\mu)}{n}} \leq 0.1$$

$$\rightarrow 2 * 1.96 * \sqrt{\frac{0.4 * (1-0.4)}{n}} \leq 0.1$$

$$\rightarrow n \geq 368.7936$$

Thus the minimum number of examples should be 369.