Polynomials and Polynomiography in Computer Science & Math

CS Seminar 198:672, Index: 62860, Semester: Spring 2012, Credits: 3

Bahman Kalantari
Professor of Computer Science, Rutgers University
Email: kalantari@cs.rutgers.edu
Web: www.cs.rutgers.edu/∼kalantar and www.polynomiography.com

Schedule: Wednesdays 10:20-1:20 PM, Hill 120 - Busch

Course Description

Polynomials are present in every branch and mathematics and the sciences with numerous fundamental applications and new ones continue to emerge. However, surprisingly there are seldom courses dedicated to their study. Even so, it would be impossible to exhaust all possible directions. This seminar offers a novel and modern point of view into theoretical and practical properties of polynomials and their applications in computer science, mathematics, education, art and more. In particular, *Polynomiography*, algorithmic visualization of polynomial root-finding, gives rise to many interesting interdisciplinary applications. We will study a subset of a wide range of topics on polynomials and polynomiography and their applications, such as:

- geometric properties of complex polynomials
- algorithms for the computation of polynomial roots, including bounds on roots
- local and global behavior of iteration functions, such as Newton's method
- topics in dynamical systems, such as Fatou and Julia sets and fractals in iteration functions
- negative results on polynomial root-finding: unsolvability, undecidability, and non-convergence
- connections to computational geometry, e.g. Voronoi diagrams
- combinatorics
- computer graphics
- approaches in visual cryptography
- educational applications, e.g. in teaching math concepts and in teaching computer algorithms
- fine art
- Smartphone applications, e.g. polynomiography-inspired games.

We believe that the study of these topics will result in fundamentally new applications and appreciation of polynomials, leading to new and worthy research topics of various kinds in computer science, mathematics, education, and many other fields. For example, not only we will offer new geometric and algorithmic results on polynomials and polynomial root-finding, but new, easy, and visually appealing proofs of classical results such as the fundamental theorem of algebra, the maximum modulus principle, the Gauss-Lucas theorem, a new interpretation of Newton's method, as well as novel polynomial root-finding algorithm that make use the ellipsoid method, giving rise to their own polynomiography.

We will also study zone diagrams, a recent variation of Voronoi diagrams, itself a well-known area of computational geometry with numerous applications. Also, a variation of zone diagrams, called mollified zone diagrams, which offers challenging research topics in computational geometry and in optimization. We will investigate possible interactions between mollified zone diagrams and polynomiography.

Another subject of study is algorithms for quaternion polynomial root-finding, applications, connections to complex polynomial root-finding, and 2D and 3D quaternion polynomiography.

Audience: This seminar is intended to appeal to a wide range of students with theoretical or practical backgrounds and interests: students in computer science, math, optimization, and other majors. While some of the underlying mathematics is advanced, the plan is to make the course appreciable by students who may not have taken courses in all related or prerequisite topics. For instance, while we will study complex polynomials, no knowledge of a complex analysis course will be needed. The course is to be self-contained in the sense that the selected topics of study will be chosen from the set of references listed below and some articles referenced therein. No textbook is to be purchased and the book listed below will be placed on reserve.

Grading: Students attending this seminar, depending upon their interest, will be asked to choose a topic, to carry out a related reading, and complement it by writing a theoretical or practical research article, or a computational implementation, or something novel, e.g. produce an actual app inspired by the course material. Students will be asked to make a presentation on their projects. A reasonably successful related research can be submitted for consideration at the upcoming ISVD conference (or a subsequent related conference): The 9th International Symposium on Voronoi Diagrams in Science & Engineering, Summer 2012 at Rutgers:
http://www.isvd12.rutgers.edu/

References

[1] T. Asano, J. Matoušek, T. Tokuyama, Zone diagrams: Existence, uniqueness, and algorithmic challenge, *SIAM Journal on Computing*, 37 (2007), pp. 1182-1198.

[2] F. Aurenhammer, Voronoi diagrams – A survey of fundamental geometric data structure, *ACM Computing Surveys* 23 (1991), pp. 345–405.

[3] S. C. de Biasi, B. Kalantari, I. Kalantari, Maximal zone ziagrams and their computation, Proceedings of the Seventh Annual International Symposium on Voronoi Diagrams in Science and Engineering (2010), pp. 171–180.

[4] S. C. de Biasi, B. Kalantari, I. Kalantari, Mollified zone diagrams and their computation, To appear in LNCS Transactions on Computational Science.

[5] B. Kalantari, *Polynomial Root-Finding and Polynomiography*, World Scientific, Hackensack, NJ, 2008.

[6] B. Kalantari, Voronoi diagrams and polynomial root-finding, Proceedings of the Sixth annual International Symposium on Voronoi Diagrams in Science and Engineering (2009), pp. 31-40.

[7] B. Kalantari, Polynomial root-finding methods whose basins of attraction approximate Voronoi diagram, Discrete & Computational Geometry, Volume 46, No 1 (2011) pp 187-203.

[8] B. Kalantari, Alternating sign matrices and polynomiography, The Electronic Jornal of Combinatorics, Volume 18, No 2 (2011), # 24.

[9] B. Kalantari, A geometric modulus principle for polynomials, The American Mathematical Monthly, December 2011. Volume 118, No 10, (2011) 931-935.

[10] B. Kalantari, Algorithms for quaternion polynomial root-finding, submitted for publication.