

CS 509: Problem Set 1

Due October 3

Rules of the game: You may collaborate with others in the class, provided that:

- You clearly state who you worked with on the problem set.
- You write up the solutions independently.

Problem 1: (Sipser, 4.22) A *useless state* in a pushdown automaton is never entered on any input string. Give a decision procedure for whether a given pushdown automaton has a useless state.

Problem 2: (*) (Sipser, 4.24) Given a DFA, M , show how to decide whether there exists a palindrome x (i.e., $x = x^R$) that is accepted by M .

Problem 3: Given a language L , let $\text{substring}(L)$ be defined by,

$$\text{substring}(L) = \{y \mid \exists x, z \text{ such that } xyz \in L\}$$

- A. Show that if L is recursively enumerable, then $\text{substring}(L)$ is recursively enumerable.
- B. Prove or disprove: If L is decidable, $\text{substring}(L)$ is decidable.

Problem 4: (Sipser, 4.18) Let A and B be two disjoint languages. Say that language C *separates* A and B if $A \subseteq C$ and $B \subseteq \bar{C}$. Show that if A and B are disjoint co-recursively enumerable sets (i.e., \bar{A} and \bar{B} are recursively enumerable and $A \cap B = \emptyset$), then there exists a decidable set C that separates A and B .

Problem 5: (*) Not all context-free grammars (CFGs) accept regular languages, but some do. How much does the size of the representation increase? Given a context-free grammar G , let $L(G)$ be the language accepted by this grammar and let $|G|$ be the “size” of G , given as the number of characters needed to write G down (the details are not important). For a language L , let $\text{min_reg}(L)$ be the number of states of the smallest deterministic finite automata accepting L , if such an automata exists, and define $\text{min_reg}(L)$ to be 0 if no such automata exists (L is not regular). Finally, define $\text{blowup}(n)$ by

$$\text{blowup}(n) = \max_{G, |G| \leq n} \text{min_reg}(G).$$

Show that $\text{blowup}(n)$ grows faster than any computable function.