CS 509: Problem Set 1

Due September 19

Rules of the game: You may collaborate with others in the class, provided that:

- You clearly state who you worked with on the problem set.
- You write up the solutions independently.

Problem 1: (Sipser 1.37) Let $C_n = \{x | x \text{ is a binary number that is a multiple of } n\}$. Show that C_n is regular for any integer constant $n \geq 1$.

Problem 2: (*)(Sipser 1.57) If A is any language, let $A_{\frac{1}{2}-}$ be the set of all first halves of strings in A so that

$$A_{\frac{1}{2}-} = \{x | \text{for some } y, |x| = |y| \text{ and } xy \in A\}.$$

Show that, if A is regular, then so is $A_{\frac{1}{2}-}$.

Problem 3: (Converse of Sipser 1.57) Prove or disprove: If $A_{\frac{1}{2}-}$ is regular, then A is regular. (This is easier than other direction, so you may wish to attempt it even if you are stumped by the other direction)

Problem 4: Suppose we have a single-counter finite state machine (it may increment or decrement a counter and may detect whether the counter is 0).

A. Show that such a class of automata can accept $a^n b^n$.

B. (*) Show that such a class of automata cannot accept ww^R.

For simplicity, assume that the counter machine is allowed to perform only a constant number of increment/decrements per input symbol read. (This limitation is not necessary.)

Problem 5: (*) Show that $L = a^n b^n$ is accepted by a two-way probabilistic finite automata with acceptance gap $\left(\frac{1}{4}, \frac{3}{4}\right)$.

To clarify some points:

- We assume that there are special symbols, say \$, that appear at the beginning and end of the input word - that is, the automata is allowed to see if it is at the end or beginning of the world.

- For a one-way finite state machine, it is standard to have “accept” and “reject” states, because it is clear when the machine is done with its computation (when it has read the last character of the input). However, this standard is awful in that it doesn’t extend to more complicated automata, particularly ones that move around on the input in both directions or have potentially large total state to manipulate (as with counter machines). The better convention is to have “accept” or “reject” be explicit actions, just like “increment counter” is an action. Thus, the specification of the counter machine might say, “If the machine is in state 3, the current symbol is ’a’, and the counter is nonzero, then stop and accept.” We do not expect you to specify your probabilistic automata in such detail!
- By an acceptance gap of $\left(\frac{1}{4}, \frac{3}{4}\right)$, we mean that if $x \notin L$, the probabilistic automata will accept with probability at most $\frac{1}{4}$, and if $x \in L$, the probabilistic automata will accept with probability at least $\frac{3}{4}$.