

## Questions: Semantic and Computational Issues

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(1) The woman that {every man/#he}<sub>i</sub> loves is his<sub>i</sub> mother.

Use properties as types to be unified? Something like:

(2)	the woman that	:	$((\text{man} \rightarrow \text{woman}) \rightarrow t) \rightarrow \text{man} \rightarrow \text{woman}$
	every man	:	$(\text{man} \rightarrow t) \rightarrow t$
	loves	:	$\text{woman} \rightarrow \text{man} \rightarrow t$
	is	:	$(\text{man} \rightarrow \text{woman}) \rightarrow (\text{man} \rightarrow \text{woman}) \rightarrow t$
	his	:	$\text{man} \rightarrow \text{man}$
	mother	:	$\text{man} \rightarrow \text{woman}$

Types need to depend on terms (for individuals and worlds): *the woman that, every man, his (man)*.

(3) Every woman<sub>j</sub> knows that the gift from her<sub>j</sub> that every son<sub>i</sub> of hers<sub>j</sub> loves is the dictionary she<sub>j</sub> gave him<sub>i</sub>.

We need notation (a type system) beyond  $\rightarrow$ .

### 1. Dependent product

(4) Types may now depend on terms.

$\text{st} : (e \rightarrow t) \rightarrow \text{type}$       (“such that”; “satisfying”)

$\llbracket \text{every} \rrbracket : \prod_{p:e \rightarrow t} (\text{st } p \rightarrow t) \rightarrow t$

(5) By analogy with product notation

$$6! = \prod_{x=1}^6 x$$

(6) By analogy with ordinary functions

$$5^7 = 7 \rightarrow 5 = \prod_{x=1}^7 5$$

(7) Subsumes ordinary functions

$$\begin{aligned} \text{st} : \prod_{p:\prod_{x:e} t} \text{type} &= (e \rightarrow t) \rightarrow \text{type} \\ \llbracket \text{every} \rrbracket : \prod_{p:\prod_{x:e} t} \prod_{q:\prod_{x:\text{st } p} t} t &= \prod p:(e \rightarrow t). (\text{st } p \rightarrow t) \rightarrow t \\ \llbracket \text{the} \rrbracket : \prod_{p:\prod_{x:e} t} \text{st } p &= \prod p:(e \rightarrow t). \text{st } p \\ \llbracket \text{that} \rrbracket : \prod_{p:\prod_{x:e} t} \prod_{q:\prod_{x:\text{st } p} t} \prod_{x:e} t &= \prod p:(e \rightarrow t). (\text{st } p \rightarrow t) \rightarrow e \rightarrow t \end{aligned}$$

(8) Typing rules for application and abstraction (cf.  $\forall$ )

$$\frac{f : \Pi x:\tau. \tau' \quad x : \tau}{f(x) : (\tau'\{x \mapsto \tau\})} \quad \frac{\tau : \text{type} \quad \begin{array}{c} [x : \tau] \\ \vdots \\ E : \tau' \end{array}}{\lambda x. E : (\Pi x:\tau. \tau')}$$

The context is not just a set of variable-type mappings anymore, but a sequence that keeps track of what we know about each variable!

Dependent sum generalizes ordinary products.

## 2. Deconstructing $\text{st}$

(9)  $t : \text{type}$

(10)  $\text{true} : t$

(11)  $\text{false} : t$

(12)  $\text{is-true} : t \rightarrow \text{type}$

(13)  $\text{is-true-indeed} : \text{is-true true}$

(14)  $\text{st} : (e \rightarrow t) \rightarrow \text{type}$

(15)  $\text{st-indeed} : \Pi p:(e \rightarrow t). \Pi x:e. \text{is-true}(p(x)) \rightarrow \text{st } p$

## 3. A few references

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