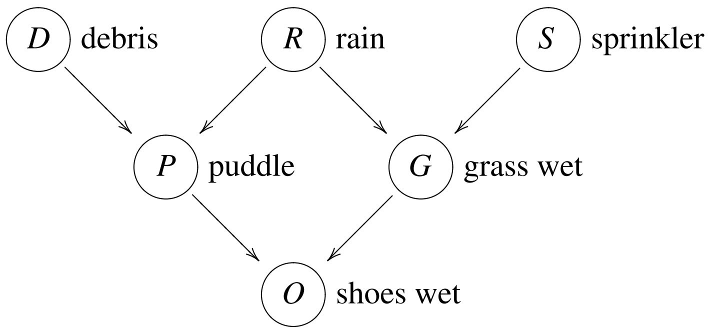
CS530 HOMEWORK 2, DUE 10/20

Please show your calculations: if you do the problems by hand or calculator, show your intermediate formulas; if you do the problems by computer, show your code.

The Bayes net below models Waldo's front yard and shoes as he leaves for work in the morning. The grass may be wet from rain or the sprinkler overnight. Rain may also form a puddle on the sidewalk, especially if there is debris. Each random variable has two possible values, true (t) and false (f).



The conditional probability tables are as follows.

D	P(D)
t	0.3
f	0.7

R	P(R)
t	0.25
f	0.75

S	P(S)
t	0.5
f	0.5

D	R	P	P($P \mid D, R$)
t	t	t	0.9
t	t	f	0.1
t	f	t	0.2
t	f	f	0.8
f	t	t	0.7
f	t	f	0.3
f	f	t	0.1
f	f	f	0.9

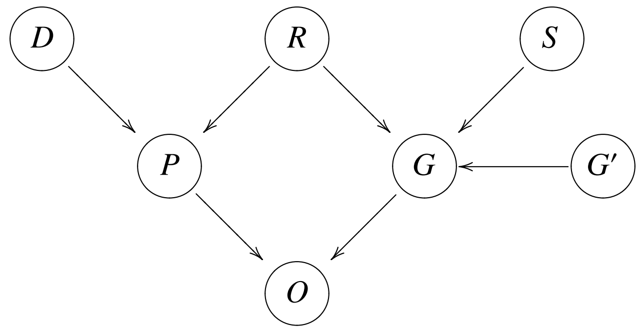
R	S	G	P($G \mid R, S$)
t	t	t	1.0
t	t	f	0.0
t	f	t	1.0
t	f	f	0.0
f	t	t	1.0
f	t	f	0.0
f	f	t	0.1
f	f	f	0.9

P	G	O	P($O \mid P, G$)
t	t	t	0.8
t	t	f	0.2
t	f	t	0.6
t	f	f	0.4
f	t	t	0.6
f	t	f	0.4
f	f	t	0.2
f	f	f	0.8

Every morning, Waldo observes whether his shoes are wet.

(1) If we were to represent the joint probability distribution of this Bayes net as a single lookup table, how many entries would it have?
(2) What is the probability that Waldo's shoes get wet?
(3) Which variables constitute the Markov blanket of G? Express as an equation the conditional independence that this Markov blanket guarantees.
(4) Suppose that Waldo observes this morning that his shoes are wet. What is the probability that the sprinkler was on?
(5) Suppose that Waldo observes this morning that his shoes are wet, and further that there is debris on the sidewalk. What is the probability that the sprinkler was on? Explain why it makes intuitive sense how this answer differs the way it does from the previous answer.

(6) In order to store and compute with this Bayes net more compactly, we can add a new random variable G', which stands for "grass wet due to some reason other than rain and sprinkler", as follows.



We can then replace the conditional probability table for $P(G \mid R, S)$ above with the deterministic statement

$$G = R \lor S \lor G',$$

where $\lor$ is just logical or. But we also need to specify $P(G')$. How? Why?

Let us move to a model of rain that is more sophisticated than just saying that the probability of rain each day is 0.25. Instead, the following table specifies the conditional probability of rain on a day given whether it rained in the previous two days.

Rain two days ago	Rain one day ago	Conditional probability of rain today
Yes	Yes	0.7
No	Yes	0.5
Yes	No	0.3
No	No	0.1

(7) Formalize this table as a hidden Markov model in which the observations are whether Waldo's shoes get wet each day. What are the states? What are the observation probabilities? What are the transition probabilities? What are the initial state probabilities? (Hint: given that Waldo has lived here for a long time, the initial state probabilities should be such that it doesn't matter which day is the first day in a sequence of days that we consider.)

On Monday and Tuesday, Waldo's shoes were dry. On Wednesday, they were wet. On Thursday, they were dry again.

(8) For each day from Monday to Thursday, compute the probability that it rained.
(9) According to the most likely sequence of states, did it rain on each day from Monday to Thursday?
(10) What is the probability that Waldo's shoes will be wet on Friday?