

Utility Guided Search of Stochastically Generated Trees

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Abstract

GAs have been found to be useful in handling many numerical optimization problems. Because of the variability in results inherent in the stochastic nature of GAs, it is common to run a GA several times and take the best of the results. However, it is possible to save a GA's population at some intermediate states and restart from one of these populations instead of from the very beginning. By doing so we generate a tree of populations, where a child population is generated from its parent by running some number of GA iterations. We describe two methods for searching such a tree of populations, one based on Highest Utility First Search (HUFFS) and one that proceeds level by level with no backtracking, and give the results of testing them on a real-world optimization task involving conceptual design of supersonic transport aircraft. They both do much better than repeatedly running the GA from the beginning, with HUFFS achieving equivalent results in less than half the GA iterations in some situations.

1 INTRODUCTION

1.1 Motivation

In many kinds of engineering design tasks, the design process involves working with candidate designs at several different levels of abstraction. E.g., in designing a microprocessor, we might start with an instruction set, implement the instructions as a series of pipeline stages, implement the set of stages as a "netlist" defining how specific circuit modules are to be wired together, etc. There are typically a combinatorially large number of ways a design at one

level can be implemented at the next level down, but only a small, fixed set of levels, perhaps a dozen or so. Thus the search space is a tree whose nodes are design alternatives, and an alternative at one level has as its children the alternatives at the next level that correctly implement it. The tree of alternatives has a small, fixed, uniform depth but a huge branching factor.

Furthermore, the partial solutions at different levels are entirely different types of things, and it is hard to come up with heuristic evaluation functions that allow us to compare, say, a set of pipeline stages with a netlist. In fact, even if S_1 and S_2 are two alternative sets of pipeline stages with S_1 better than S_2 by the standard metrics, it is possible that there are netlists N_1 and N_2 descended, respectively, from S_1 and S_2 such that N_1 is clearly worse than N_2 . It even possible, though less likely, that the *average* descendant of S_1 is worse than the average descendant of S_2 . The huge branching factor and the limitations in comparing designs within and across levels make it hard to apply standard tree-search algorithms to guide the search for a design.

Recently, a number of techniques for stochastic optimization have been shown to be useful for such problems. These techniques include simulated annealing [10; 27], genetic algorithms [15; 8; 18; 1], and random-restart hill climbing [28]. A design at one level is translated into a correct but poor design at the next level, and a stochastic optimizer is used to improve this design. An inherent feature of a stochastic method is that it can be run again and again on the same inputs, each time potentially producing a different answer. These alternatives can each be used as inputs to a similar process at the next lower level. Thus, these optimizers can be seen as generating a tree of design alternatives. These trees are much smaller than the original trees, and consist only of relatively high-quality alternatives. However, there can still be significant varia-

tions in quality among alternatives and these trees can still have a large branching factor (in the thousands for examples we have looked at). Also, it can take from minutes to days of computer time to run an optimizer and generate a single descendant. So, there is still the problem of controlling the search within the smaller tree, that is, for deciding which alternative to generate a child from next. We will refer to such trees as “Stochastically Generated” trees.

Finding the globally optimal design in these kinds of problems is computationally intractable, so our goal is not a search process which finds the best design but one which has the best *tradeoff* between the quality of the result we get and the cost in search time it takes to find that result. In decision theoretic terms [13; 26] we are looking for the search process with the highest expected utility.

In prior work we developed an algorithm for searching Stochastically Generated trees called Highest Utility First Search (HUFs) [23]. As the name implies, HUFs works by estimating the utility of continuing the design search in different parts of the tree of alternatives, and focusing on those parts with the highest utility.

This paper will discuss three new developments:

- HUFs has been applied to and tested on a design problem that involves both a task domain (aircraft design) and an underlying optimizer (a Genetic Algorithm) that are quite different in nature from the original test domain and optimizer (placement of digital circuits and a random-restart hill climber, respectively). In the new problem HUFs has shown gains in search efficiency over alternative methods that are similar to the gains it showed in the original test problem.
- This work has shown the usefulness for Genetic Optimization of maintaining and searching a tree of alternative populations of solutions, rather than keeping only a single current population as is typically done.
- The model of utility used by HUFs has been generalized. The most important aspect of this generalization is that it allows for a deadline – a time by which a solution *must* be produced if it is to have any utility at all.

We will first discuss the new test problem and the optimizer and the notion of searching a tree of population. The section after that will describe the new model of utility and the HUFs algorithm as it has been revised to use this new model. (The reader will not be assumed to be familiar with previous work.) Next, we will describe the empirical testing we have done on the revised algorithm. The final two sections will contain a discussion of related work and the summary and conclusions.

1.2 The Test Problem

Our test problem concerns the conceptual design of supersonic transport aircraft. We summarize it briefly here;

Table 1: Aircraft Parameters to Optimize

No.	Parameter
1	exhaust nozzle convergent length(l_c)
2	exhaust nozzle divergent length(l_d)
3	exhaust nozzle external length(l_e)
4	exhaust nozzle radius(r_7)
5	engine size
6	wing area
7	wing aspect ratio
8	fuselage taper length
9	effective structural t/c
10	wing sweep over design mach angle
11	wing taper ratio
12	Fuel Annulus Width

it is described in more detail elsewhere [6]. Figure 1 shows a diagram of a typical airplane automatically designed by our software system. The system attempts to find a good design for a particular mission by varying twelve of the aircraft conceptual design parameters in Table 1 over a continuous range of values.

The underlying system that HUFs is to control combines a Genetic Algorithm (GA) based optimizer with a multidisciplinary aircraft simulator. In our current implementation, the optimizer’s goal is to minimize the takeoff mass of the aircraft, a measure of merit commonly used in the aircraft industry at the conceptual design stage. Takeoff mass is the sum of fuel mass, which provides a rough approximation of the operating cost of the aircraft, and “dry” mass, which provides a rough approximation of the cost of building the aircraft. The fuel mass needed is computed by simulating the aircraft’s flight over a specified “mission”, which says how far the aircraft flies at each of several altitudes and speeds. Calculating takeoff mass for one candidate design requires about 0.2 CPU seconds on a DEC Alpha 250 4/266 desktop workstation.

The aircraft simulation model used is based on both implicit and explicit assumptions and engineering approximations. Since it is being used by a numerical optimizer rather than a human domain expert, some design parameter sets may correspond to aircraft that violate these assumptions and therefore may not be physically realizable even though the simulator does not detect this fact. We refer to these designs as *infeasible points*. For this reason a set of constraints has been introduced to safeguard the optimization process against such violations. Other constraints enforce requirements such as the requirement that there be room for some specified number of passengers inside the aircraft. The result of optimization must satisfy all of the constraints, as well as having a low takeoff mass.

We generate a range of different problems by varying two parameters: the percentage of the mission that is to be flown at subsonic speeds (to avoid sonic booms over populated areas), and how many passengers the aircraft must accommodate. A problem specification then consists

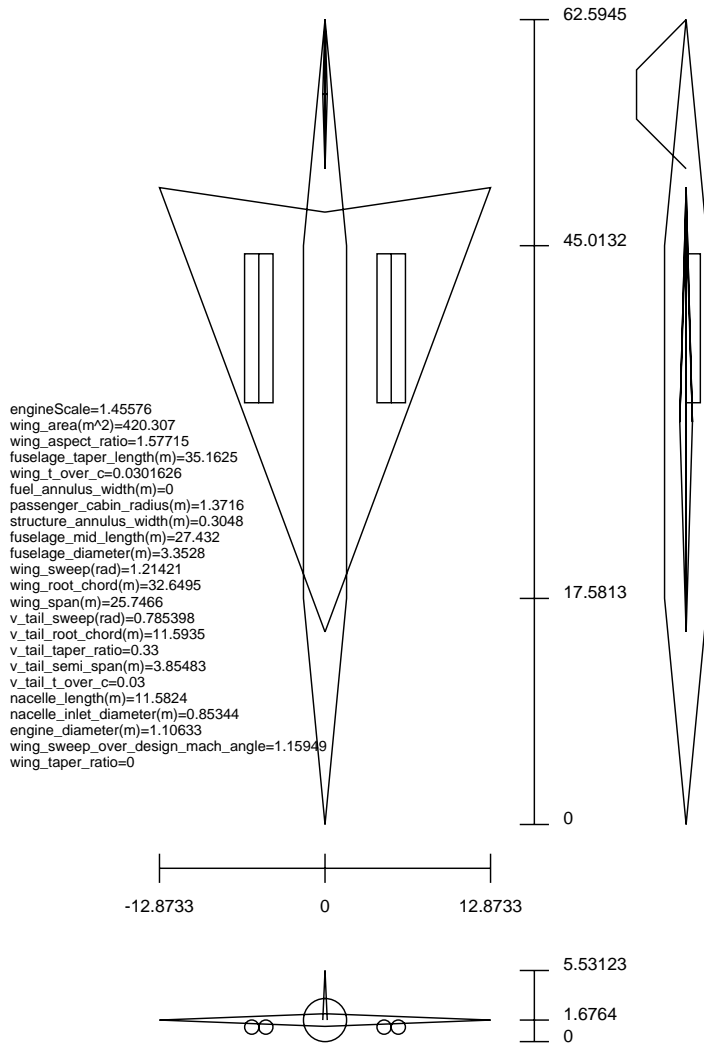


Figure 1: Supersonic transport aircraft designed by our system (dimensions in feet)

of values for these two numbers.

The optimizer used was GADO [18; 19], a GA that was developed with the goal of being suitable for use in engineering design. It uses new operators and search control strategies that target the domains that typically arise in such applications. GADO has been applied in a variety of optimization tasks that span many fields, and has demonstrated a great deal of robustness and efficiency relative to competing methods.

In GADO, each individual in the GA population represents a parametric description of an artifact, such as an aircraft or a missile. All parameters take on values in known continuous ranges. Floating point representation is used. The fitness of each individual is based on the sum of a proper measure of merit computed by a simulator or some analysis code (such as the takeoff mass of an aircraft), and a penalty function that is a function of the number and magnitude of constraint violations. A steady state GA model is used, in which operators are applied to two parents selected from the elements of the population via some

selection scheme, one offspring point is produced, then an existing point in the population is replaced by the newly generated point via some replacement strategy. Here selection was performed by rank because of the wide range of fitness values caused by the use of a penalty function. The replacement strategy used here is a crowding technique, which takes into consideration both the fitness and the proximity of the points in the GA population. The population size is an external parameter with a default value of 10 times the dimension of the search space. For the aircraft problem we used the default, giving 120 individuals in the population. The initial population is generated by randomly generating many individuals and using the replacement strategy to keep the population at 120.

Several crossover and mutation operators are used, most of which were designed specifically for numerical optimization problems of this type. GADO also uses a search-control method that saves time by avoiding the full evaluation of points that are unlikely to correspond to good designs. The GA stops when either the maximum number

of evaluations has been exhausted or the population loses diversity and practically converges to a single point in the search space.

1.3 Trees of Populations

A genetic algorithm maintains a set, or “population” of “individuals”. In our case, each individual is a candidate solution to our optimization problem. New individuals are added by modifying or combining existing individuals in a process that is randomized but weighted in favor of using good solutions (those with low takeoff weight and lesser constraint violations) as “parents” of the new individual. Individuals are removed from the population by a process that is weighted in favor of removing poor solutions. The best individual created during a run becomes the result of the optimization.

Because of the random component of the process, if you run a GA repeatedly on the same problem you will get a range of different results, and these results will vary in quality, i.e. in the measure of merit. Even though this variability is often less than with competing methods like gradient descent, it is common to run a GA a few times on any given problem, and to take the best of the resulting answers as the overall answer to the problem.

Current practice is to simply run the entire optimization process several times from beginning to end. Suppose, however, that we ran the GA in stages, stopping after every n iterations and saving the current population. We can view the process of running for n iterations as an operator that generates one population from another. Because of the stochastic nature of this operator, we can apply it to the same “parent” population (that is, go back and restart from the same saved population) many times, and it will generate many different “child” populations. We can also apply the operator to one or more child populations. In this way we can generate a tree of populations.

We can view the optimization process as searching this tree of populations. The basic step is to start with some population and apply the operator (i.e. run the GA for n iterations), to generate one child. When we go back we need not go back to the very beginning, and when we run the GA we need not run to convergence. Rather, each time we do a step we can choose any existing population as our starting point and take one step from there.

GADO calls the simulator to evaluate a candidate about 12,000 times in order to converge on an answer to the aircraft design problem, including 3,600 calls in the initialization phase. We have chosen to divide this into five chunks, a first group of 4,000 iterations to get past the initialization, and four more chunks of 2,000 iterations each. Thus, the tree to be searched has a root representing the initial state with problem specifications but no individuals generated yet, and five levels whose nodes are populations after, respectively, 4, 6, 8, 10, and 12 thousand calls to the evaluator.

2 Searching the Tree of Populations

Searching a tree is one of the classic tasks in Computer Science in general. However, the tree search problem here is somewhat different from the problems we normally think of.

- The tree is quite shallow, with a depth of at most 5, but the branching factor is combinatorially large - it is the number of different populations we could possibly arrive at in 2,000 (or 4,000) iterations.
- the only way we have to generate children from a node is a stochastic operator that, each time it is applied, randomly selects which of the possible children it will return. Furthermore, this operator takes five to ten minutes to apply, so we can afford a fairly large amount of computing per operator application for reasoning about control of the search.
- While we care about the total time taken, and thus the total number of operator applications, we do not care about how long the path in the tree is from root to the solution. This is in contrast to planning problems where the path in the tree represents a sequence of steps to be carried out so shorter paths are preferred.

Because of these differences, some search algorithms, e.g. A^* , are irrelevant (since we do not want to optimize path length) and/or impossible (since we cannot practically generate all children of a node). Furthermore, finding the absolute global optimum design almost always takes more computing time than we can afford, so we need to trade off computing time for result quality. One search control method that applies naturally in this situation is Highest Utility First Search (HUFS) [23]. In this section we will first discuss the notion of utility in general and the particular formulation we used, then we will discuss HUFS, and then we will describe a much simpler search method, Waterfall, that we also tested.

2.1 Utility and Satisficing

The notion of utility comes originally from the field of decision theory. A decision results in some outcome, and the utility of an outcome is the overall net benefit of this outcome. The utility of an action is the average utility of the outcomes that may result from the action, weighted by their probability of occurring. In the literature on time-bounded computing. e.g., [22], the utility of computing some result r is seen as a function $U(r, t)$ of the result itself and the time, t , at which it becomes available. By reasoning about the expected utility of alternative actions, a control method can handle the tradeoff between compute time and result quality.

In much of the literature, U is further assumed to be of the form

$$U(r, t) = I(r) - C(t)$$

where $I(r)$ is some intrinsic value in having r , and $C(t)$ represents a “time cost”, usually a linear function of t , i.e. a constant cost per unit of time. For our purposes, however, this formulation has a major drawback in that it does not easily handle deadlines. In most real world engineering situations, there is some deadline by which a design simply must be produced; a design that is produced later, no matter how good it is intrinsically, is worth nothing. We believe that most real world engineering design also has a satisficing character. The designer has in mind some goal level of design quality which is “good enough”, and the primary aim of the designer is to produce a design that is good enough (meets or exceeds the goal level of quality) and to do so soon enough (before the deadline). Thus, we assume there is some deadline, and formulate the utility of a result as a simple threshold function on its quality and the current time:

$$U_r(Q(r), t) = \begin{cases} 1 & \text{if } Q(r) \leq Q_g \text{ and } t \leq t_g \\ 0 & \text{otherwise} \end{cases}$$

where Q is some quality metric (e.g., takeoff mass), where Q_g is the desired quality and t_g is time the deadline occurs. (Note that lower Q is better.) Then the utility of an action becomes simply the probability that it will result in a design with quality of Q_g or better by time t_g . This is of course also a simplification of the real world, but we believe it is a better model than using a fixed cost per unit time.

2.2 HUFs

HUFs works by estimating, for each possible parent, the expected utility of the design that would result if the search were restricted to the given parent and its descendants. At each step HUFs chooses the parent that currently has the best estimated utility and generates one child from that parent. The score of the new child is used to update the utility estimates, and the process repeats. In this section we will describe how these expected utilities are estimated. We start by discussing the idea of a *Child Score Distribution*.

2.3 Scores and Child Score Distributions

We assume that we have some heuristic evaluation function $S(a)$ that assigns a numerical score to a population a . The score of a population is an estimate of how good a result we will get if we use it as our starting point. For simplicity we have defined $S(a)$ to be the quality metric of the best individual result in a . That is, viewing population a as a set of candidate results, since lower Q is better,

$$S(a) = \min_{r \in a} Q(r)$$

This means that if we stop searching and take the best result r' in a as our answer to the optimization problem, then $Q(r') = S(a)$. So, our search succeeds if we find a population with $S(a) \leq Q_g$ by time t_g .

If we generate a child (i.e., run the GA a specified number of iterations and generate a new population) and calculate its score, the value we get depends to some extent on the randomized choices made by the GA while generating the child, so we can treat the score as a random variable. Since generating a child does not change the parent, the score of one child has no effect on the score we will get if we generate another child from the same parent. Therefore for a given parent, a , we can view the scores of its children as independent variables with identical distributions. We define the *Child Score Distribution*, G_a of a parent, a , to be this probability distribution of its childrens' scores. That is,

$$G_a(s) = P(S(b) = s | b \in \hat{C}(a))$$

where $\hat{C}(a)$ is the set of populations that might be generated as children of a .

We assume that G_a is a normal distribution, and that the mean and standard deviation of G_a are themselves randomly drawn, respectively, from normal distributions M and D . We assume the standard deviations of M and D are constants and that their means are linear functions of the score $S(a)$. That is, we assume

$$M(\mu) = P(\text{mean}(G_a) = \mu) = Z(\mu, x * S(a) + y, d)$$

where $Z(\mu, x * S(a) + y, d)$ is the Gaussian “bell curve” function with mean $x * S(a) + y$ and standard deviation d , calculated at point μ , and where x , y , and d are constants. We make a similar assumption about D . The constants are estimated for each level of the tree by some initial exploration. This gives us an a priori estimate of G_a based on $S(a)$, and we update the estimate as we generate children from a and thus sample the actual distribution. The assumptions give only crude approximations to the true distributions, but as will be seen below the approximations are good enough to allow HUFs to perform well in practice.

This is similar to the approach to estimating child score distributions in [23]; please see that paper for a fuller discussion.

2.4 Estimating Utilities

Next, we will describe how the Child Score Distributions are used to estimate the utility of searching under a given parent. In doing so, we will use the following notation:

- As above, $\hat{C}(a)$ is the set of child populations that *can* be generated, while $C(a)$ is the set of children that *have* been generated already.
- $S_{\min}(a) = \min_{c \in C(a)} S(c)$, i.e. it is the minimum score of any child of a . If a has no children, $S_{\min}(a) = \infty$.
- τ is the time needed to generate one child.
- $U_r(q, t)$ is the expected utility of having a result with quality q by time t

- $U_p(a, t)$ is the utility of searching under parent a starting the search at time t .
- $U_p(a, t|C(a))$ is the expected value of $U_p(a, t)$ given some condition $C(a)$. E.g., $U_p(a, t|S(a) = s)$ is the expected value of $U_p(a, t)$ given that a 's score is s .
- We number levels in the tree upward from the lowest level, level 0, to the root (level 5), and use superscripts to denote the level. So $U_p^1(a, t|S(a) = s)$ is the utility of searching under a population a at level 1 whose score is s .

The utility of searching under a parent of course depends on the search algorithm used, and the utility estimates themselves are part of the search algorithm. We simplify this self-referential problem by assuming a restricted search algorithm in our utility analysis: to search under parent a at level i in the tree, generate children of a until we find a child c whose estimated U_p is larger than that of the a , that is, until

$$U_p^{i-1}(c, t) > U_p^i(a, t)$$

where t is the current time. then apply this algorithm recursively to c . Stop when you find a child whose score is less than (i.e. better than) Q_g .

This restricted version ignores the possibility that after we switch to c and generate some of *its* children, we may revise our estimated utilities and decide that some sibling of c , or perhaps even a itself, has a higher utility. In that case the actual HUFs algorithm would generate the next child from the parent with the currently-highest U_p , while the restricted algorithm would not.

To estimate $U_p^i(a, t)$, where a is the parent we will work recursively on both time and level number. There are two base cases.

- If $t_g < t$, i.e. the deadline has passed, all utilities are 0.

$$\text{if } t_g < t \text{ then } U_p(a, t) = 0$$

- If a is on level 0, we do not generate any further children, so all we can do is take the best result in a as our answer, and

$$\begin{aligned} U_p^0(a, t) &= U_r(S(a), t) \\ &= 1 \text{ if } S(a) \leq Q_g \text{ and } t \leq t_g, \\ &\quad \text{else } 0 \end{aligned}$$

In general, $U_p^i(a, t)$ depends on $S(a)$, $G(a)$, and, if a has any children, on $\min_{c \in C(a)} S(c)$.

- If we choose to stop and use the best result in a as our answer, then as above

$$U_p^i(a, t) = U_r(S(a), t)$$

- If we choose to switch to generating children from a child of a , we will choose the child with the best utility, and the utility of our search becomes that child's utility. But, by the restrictions on the algorithm discussed above, we have not generated any children from any child of a , so all we know about a 's children are their scores. The best child is then one with the lowest score, so

$$U_p^i(a, t) = U_p^{i-1}(c, t|S(c) = S_{\min}(a))$$

Note that it takes no time to switch so we still use time t .

- If we choose to generate another child from a , it will be available at time $t + \tau$. It will have score s with probability $G_a(s)$, but it will only change $U_p^i(a, t + \tau)$ if it is better than the best child of a we already have, i.e. has a lower score, so the expected utility is

$$\begin{aligned} U_p^i(a, t) &= \\ &\int_0^{S_{\min}(a)} G_a(s) U_p^{i-1}(c, t + \tau|S(c) = s) ds \\ &+ U_p^{i-1}(c, t + \tau|S(c) = S_{\min}(a), t) \\ &\quad * \int_{S_{\min}(a)}^{\infty} G_a(s) ds \end{aligned}$$

In fact, we will choose the one of these three alternatives with maximum utility, so $U_p^i(a, t)$ is the maximum of these three possible values. Note that where we recur it is on a lower level or a later time, so the recursion terminates.

2.5 Waterfall

As a check on whether we needed the complexity of HUFs, we also tried a very simple search algorithm, the Waterfall algorithm from [23]. This algorithm works from the top down. At each level it takes the population with the lowest (best) score and generates a fixed number of children from it, then it chooses the best of those children and repeats.

3 Empirical Test

The empirical test was run on the aircraft optimization problem described above. Rather than use CPU time for t and τ we counted iterations of GADO. The implementation uses C and LISP, and runs on SUN workstations. Generating a population takes 5-10 minutes and running HUFs takes up to a minute or so per population.

We first created 10 random problems (combinations of number of passengers and percent supersonic) for calibration. On each problem we generated a population at each level of the tree and generated 10 children for each of these. From the scores of these populations we estimated the parameters for the a priori estimates of $G(a)$ as a function of $S(a)$.

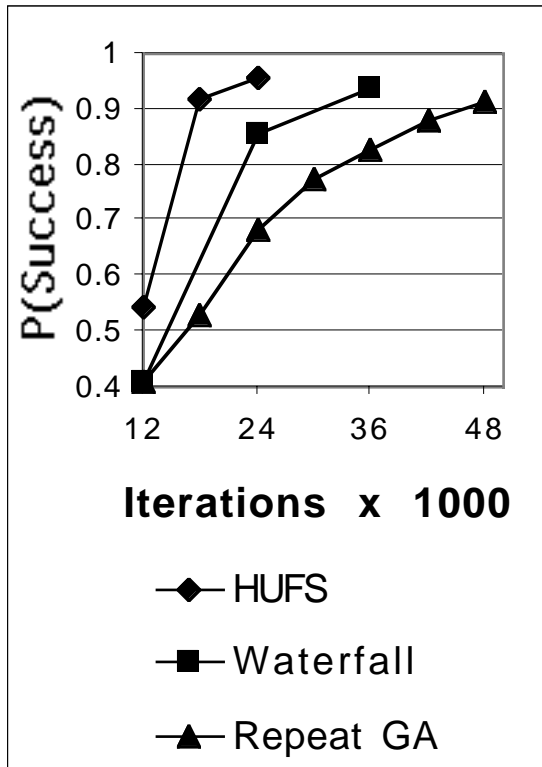


Figure 2: P(Success) averaged over 4 test problems

Next we created 4 more problems for testing. We then ran HUFS and Waterfall 12 times on each and also ran plain GADO 12 times on each problem, for each of several different t values. Since repeated runs of GADO are independent, we simulated the process of repeating GADO by repeatedly randomly choosing a run from the 12 we had done on a given problem. We did this 1000 times for each combination of problem and t .

The Q_g was set for each problem so that repeating GADO would take about 48000 iterations to have a 90% chance of success, i.e. of achieving Q_g .

Results are shown in Figure 2. It can be seen that if it is worthwhile doing extra GADO iterations (beyond the 12,000 needed for one full run of GADO) in order to improve the probability of success, then doing a tree search guided by HUFS can get the same probability of success in half the GADO iterations or less compared to simply repeating GADO over and over.

Waterfall's performance is also much better than that of repeated GADO, but not as good as HUFS. On the other hand, HUFS is much more complex.

4 Related Work

Several research efforts have applied genetic algorithms to engineering optimization and search problems in a variety of domains, including control system design [11], architectural and civil engineering design [7; 21], VLSI design [12], mechanical design [2] and aircraft design [16]. Deb [4; 3]

developed a GA called GeneAS for engineering design optimization with mixed variables (both discrete and continuous). He demonstrated the merit of his GA in the domain of mechanical component design. Powell [17; 24] has built a module called Inter-GEN, part of the ENGINEOUS system [25]. It contains a genetic algorithm and a numerical optimizer, and uses a rule-based expert system to decide when to switch between the two. Powell tested his system on a realistic design task (jet engine design). Combining GAs and knowledge-based systems was also done in [20].

Several research efforts have focused on preventing premature convergence in GA search. An important class of methods are replacement strategies that take into consideration other factors in addition to fitness (such as preserving diversity in the population for example). Each such strategy is called a *crowding heuristic* and the reader is referred to [14] for a detailed discussion of these methods. Another class of methods focus on carefully choosing the population size and the reader is referred for example to [9].

Several research efforts have used utility based decision making to improve design optimization, e.g., [22] and [5]. In particular, HUFS is very much in the spirit of Russell and Wefald's LDTA* algorithm [22] which uses utility estimates to guide the search of a classical problem-solving search trees and learns the probability distributions it needs for estimating utilities at the same time as it is doing the search. The method LDTA* uses to estimate utilities and to guide the search are quite different from those of HUFS because of the different character of the search problems the two systems control. Similarly, the two systems estimate different distributions - LDTA* estimates the distribution of errors in a prediction based on a heuristic score while HUFS estimates the distribution of scores - and they use different methods for learning the distributions.

To the best of our knowledge, no research efforts have attempted to combine utility based decision making with GA search in a way similar to the proposed approach, nor has any previous work looked at searching the space of GA populations.

5 Conclusions

We have demonstrated that HUFS improves significantly over Waterfall in a problem that is very different from the original module placement problem used in the development of HUFS. That problem is combinatorial in nature, while this one involves a more-or-less continuous system. The optimizers are also different. We have also added a data point regarding usefulness of HUFS even when the tree being searched is not made up of design abstraction levels. Rather, HUFS seems to apply more generally to trees with small, limited depth, combinatorially large branching factors, and an operator that generates random children.

We have extended HUFS to work with utility functions with deadlines.

We have also demonstrated that, in a realistic engineering optimization problem, it can be useful to think of a GA as generating a virtual tree of populations, and to explicitly search this tree, either with HUFFS or Waterfall as the search control method. These results clearly need to be replicated on additional problems and with other GAs. We believe that the idea of searching the space of populations, rather just using the GA's operators to generate a sequence of populations, will be a very fruitful one to explore.

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