Algorithmic Information, Fractal Geometry, and Decentralized Dynamics

Neil Lutz

Dept. of Computer Science

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Abstract

This dissertation applies two distinct algorithmic perspectives to questions in the field of fractal geometry and dynamics.

In Part I, we establish connections between algorithmic information theory and classical fractal geometry. Working in Euclidean spaces, we characterize Hausdorff and packing dimensions in terms of relativized Kolmogorov complexity, and we develop conditional dimensions. These tools give rise to new dimensional bounding techniques, which we apply to problems in fractal geometry. Most significantly, we prove that a classical dimension bound for intersections of Borel sets holds for arbitrary sets, and we give a new lower bound on the Hausdorff dimension of generalized Furstenberg sets.

In Part II, we use ideas from distributed computing and game theory to study dynamic and decentralized environments in which computational nodes interact strategically and with limited information. We exhibit a general non-convergence result for a broad class of dynamics in asynchronous settings. For uncoupled game dynamics, in which preferences are private inputs, we give new bounds on the recall necessary for self stabilization to an equilibrium.

Defense Committee: Rebecca Wright (Chair), Eric Allender, and Shubhangi Saraf