Abstract

We characterize the streaming space complexity of every symmetric norm (a norm on $\mathbb{R}^n$ invariant under sign-flips and coordinate-permutations), by relating this space complexity to the measure-concentration characteristics of $x$. Specifically, we provide upper and lower bounds on the space complexity of approximating the norm of the stream, where both bounds depend on the median and maximum of $(x)$ when $x$ is drawn uniformly from the 2 unit sphere. The same quantity governs many phenomena in high-dimensional spaces, such as large-deviation bounds and the critical dimension in Dvoretzky’s Theorem. The family of symmetric norms contains several well-studied norms, such as all $p$ norms, and indeed we provide a new explanation for the disparity in space complexity between $p^2$ and $p>2$. In addition, we apply our general results to easily derive bounds for several norms that were not studied before in the streaming model, including for example the top-k norm and the k-support norm, which was recently shown to be effective for machine learning tasks.