Abstract

It is known from the works of Lovett-Meshulam-Samrodinitsky, Tao-Ziegler, and more recently, Bhowmick-Lovett, that there are Boolean functions which do not correlate well with classical polynomials of a certain degree but have good correlation with some non-classical polynomial of the same degree.

Noting that the notion of approximation is different from that of correlation in the case of non-classical polynomials, Bhowmick and Lovett asked the following questions:

* Do non-classical polynomials of degree \( \sqrt{n} \) approximate the majority function better than classical polynomials of the same degree?

* Is there any Boolean function for which non-classical polynomials offer an advantage over classical polynomials in the case of approximation?

We give a negative answer to the first question. We do so by studying polynomials over rings of the form \( \mathbb{Z}/p^k\mathbb{Z} \) and observing that non-classical polynomial are a special case of such polynomials. Our proof essentially involves proving bounds for "weak representations" of the majority function over \( \mathbb{Z}/p^k\mathbb{Z} \), strengthening classical results of Szegedy and Smolensky.

For the second question, we give a positive answer by showing that elementary symmetric polynomials of a suitable degree are well approximated by non-classical polynomials.

Joint work with Prahladh Harsha and Srikanth Srinivasan.