Abstract

We characterize the streaming space complexity of every symmetric norm (a norm on \(\mathbb{R}^n\) invariant under sign-flips and coordinate-permutations), by relating this space complexity to the measure-concentration characteristics of \(x\). Specifically, we provide upper and lower bounds on the space complexity of approximating the norm of the stream, where both bounds depend on the median and maximum of \((x)\) when \(x\) is drawn uniformly from the 2 unit sphere. The same quantity governs many phenomena in high-dimensional spaces, such as large-deviation bounds and the critical dimension in Dvoretzky’s Theorem. The family of symmetric norms contains several well-studied norms, such as all \(p\) norms, and indeed we provide a new explanation for the disparity in space complexity between \(p^2\) and \(p>2\). In addition, we apply our general results to easily derive bounds for several norms not studied before in the streaming model, including for example the top-k norm and the k-support norm, which was recently shown to be effective for machine learning tasks.

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