1 Problem - λ-term abbreviations

Give the fully expanded λ-terms for the following λ-term abbreviations as discussed in class (lecture 6, page 5).

1. \((fgx)v\) w
2. \(\lambda xy.(\lambda z.x(yz))v\) w
3. \(\lambda x.xx\) \(\lambda x.xx\)

2 *Problem - β-reductions

Show all possible β-reduction sequences for the λ-term

\((\lambda xyz.(xz)(yz)) (\lambda xy.x) (\lambda xy.x)\)

Clearly mark the redex at each step (see lecture 6, pages 17, 18)

3 Problem - Normal forms

1. Give three distinct λ-terms that reduce to the same normal form
2. Give three distinct λ-terms that do not have a normal form
3. Give a λ-term for which the normal form and the head-normal form are distinct.

4 Problem - Church-Rosser property

Using CR-I as discussed in class (see lecture 6, page 19), prove that if a normal form exists for λ-term M, it has to be unique.
5  *Problem - Programming in lambda calculus*

Logical constants and operations can be represented in lambda calculus as follows:

\[
\begin{align*}
\text{true} & \equiv \lambda a. \lambda b. a \\
\text{false} & \equiv \lambda a. \lambda b. b \\
\text{not} & \equiv \lambda x. ((x \text{ false}) \text{ true}) \\
\text{and} & \equiv \lambda x. \lambda y. ((x y) \text{ false}) \\
\text{or} & \equiv \lambda x. \lambda y. ((x \text{ true}) y)
\end{align*}
\]

Give the lambda calculus implementation of the boolean operations:

1. **implication** \( a \to b \)
2. **exclusive or** \( a \text{ exor } b \)