Class Announcements

Here is where we are:

- Second project due Monday, April 24
- Third project has been posted; due Monday, May 1
- Sixth homework will be posted later today or tomorrow; due Tuesday, April 25
- Seventh homework will be posted next week; due Monday, May 1
- Final (third midterm) exam on Thursday, May 4, noon - 3:00pm timeslot. Location: most likely this room.

  Any CONFLICTS with other classes?

Dependence — Overview

Definition — There is a data dependence from statement S_1 to statement S_2 ($S_1 \delta S_2$) if

1. Both statements access the same memory location, and
2. There is a run–time execution path from S_1 to S_2.

Data dependence classification

"S_2 depends on S_1" — $S_1 \delta S_2$

true (flow) dependence
: occurs when S_1 writes a memory location that S_2 later reads

anti dependence
: occurs when S_1 reads a memory location that S_2 later writes

output dependence
: occurs when S_1 writes a memory location that S_2 later writes

input dependence
: occurs when S_1 reads a memory location that S_2 later reads. Note: Input dependences do not restrict statement (*load/store*) order!

Dependence Analysis

Question

Do two variable references never/maybe/always access the same memory location?

Benefits

- improves alias analysis
- enables loop transformations

Motivation

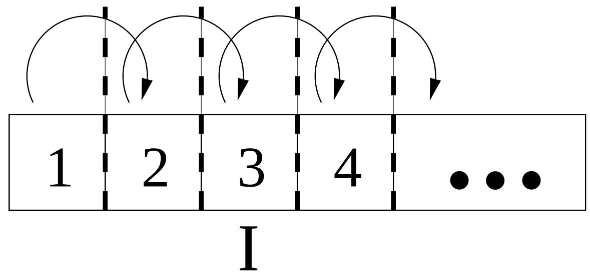
- classic optimizations
- instruction scheduling
- data locality (register/cache reuse)
- vectorization, parallelization

Obstacles

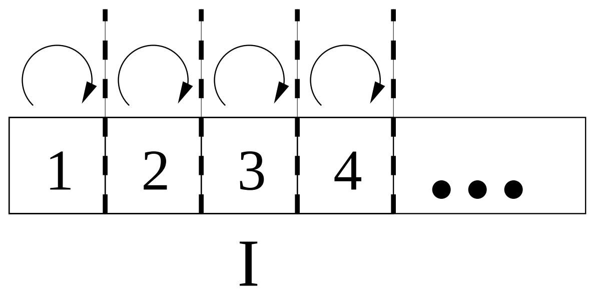
- array references
- pointer references

Dependence Analysis for Array References

```
do I = 1, 100
   A(I) =
      = A(I-1)
enddo
```




```
do I = 1, 100
   A(I) =
      = A(I)
enddo
```



A **loop-independent** dependence exists regardless of the loop structure. The source and sink of the dependence occur on the same loop iteration.

A **loop-carried** dependence is induced by the iterations of a loop. The source and sink of the dependence occur on different loop iterations.

Loop-carried dependences can inhibit parallelization and loop transformations

Dependence Testing

Given

```
do i_1 = L_1, U_1
   ...
     do i_n = L_n, U_n
S1     A(f_1(i_1,...,i_n),...,f_m(i_1,...,i_n))  =  ...
S2     ...  =  A(g_1(i_1,...,i_n),...,g_m(i_1,...,i_n))
```

A *dependence* between statement S_1 and S_2, denoted $S_1 \delta S_2$, indicates that S_1, the *source*, must be executed before S_2, the *sink* on some iteration of the nest.

Let α & β be a vector of n integers within the ranges of the lower and upper bounds of the n loops.

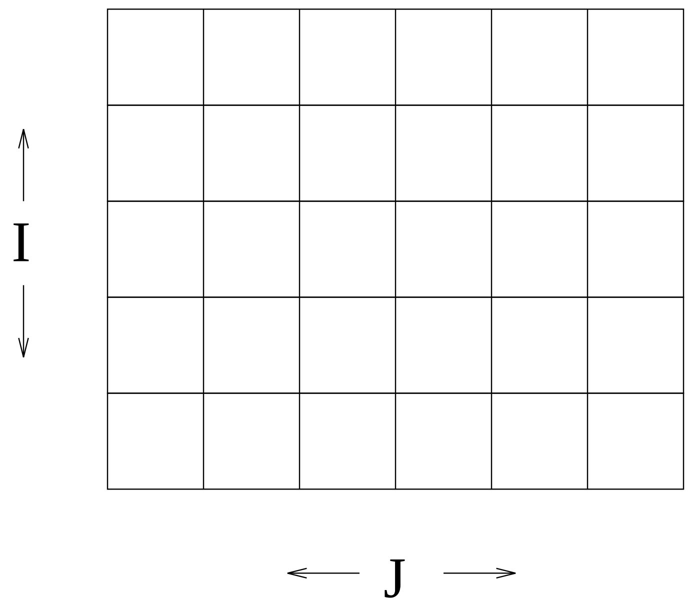
Does $\exists\ \alpha \leq \beta$, s.t.

$$f_k(\alpha) \;=\; g_k(\beta) \quad \forall k,\ 1 \leq k \leq m\ ?$$

Iteration Space

```
do I = 1, 5
   do J = I, 6
      ...
   enddo
enddo
```

1 $\leq$ I $\leq$ 5
I $\leq$ J $\leq$ 6



- lexicographical (sequential) order for the above iteration space is

  (1,1), (1,2), . . ., (1,6)
  (2,2), (2,3), . . . (2,6)
  . . .
  (5,5), (5,6)

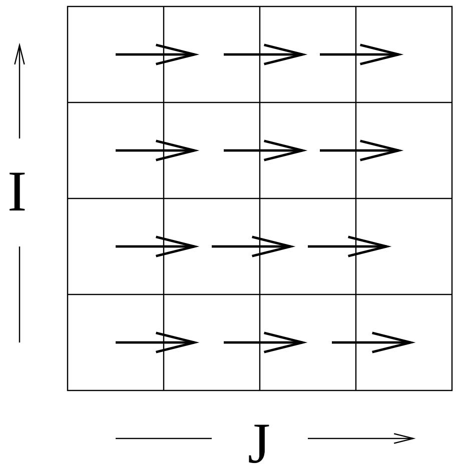
- given $I = (i_1, \ldots i_n)$ and $I' = (i'_1, \ldots, i'_n)$,

  $I \; < \; I'$ *iff*

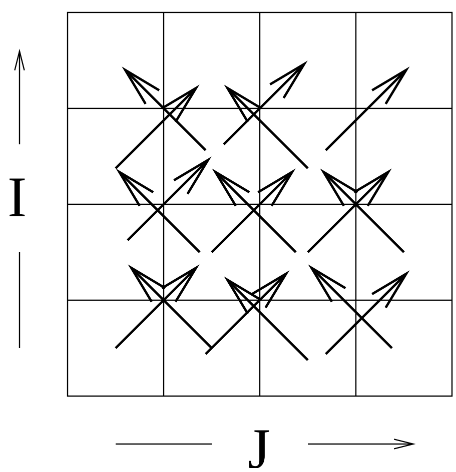
  $(i_1, i_2, \ldots i_k) = (i'_1, i'_2, \ldots i'_k) \;\; \& \;\; i_{k+1} < i'_{k+1}$

Distance & Direction Vectors

```
 do I = 1, N
  do J = 1, N
S1 A(I,J) = A(I,J-1)
  enddo
 enddo
```




```
 do I = 1, N
  do J = 1, N
S2 A(I,J) = A(I-1,J-1)
S3 B(I,J) = B(I-1,J+1)
  enddo
 enddo
```



Distance Vector = number of iterations between accesses to the same location

Direction Vector = direction in iteration space $(=, <, >)$

	distance vector	direction vector
$S_1 \delta S_1$	$(0, 1)$	$(=, <)$
$S_2 \delta S_2$	$(1, 1)$	$(<, <)$
$S_3 \delta S_3$	$(1, -1)$	$(<, >)$

Approaches to Dependence Testing

- can we solve this problem exactly?
- what is conservative in this framework?
- restrict the problem to consider index and bound expressions that are linear functions

$\Longrightarrow$ solving general system of linear equations
in integers is NP-hard

Solution Methods

- inexact methods
  - Greatest Common Divisor (GCD)
  - Banerjee's inequalities
- cascade of exact, efficient tests
  (fall back on inexact methods if needed)
  - Rice (see posted PLDI'91 paper)
  - Stanford
- exact general tests (integer programming)

Dependence Testing

SIV - Single Induction Variable Test

1. Single loop nest with constant lower (LB) and upper (UB) bounds, and step 1

   ```
   for i = LB, UB, 1
    ...
   endfor
   ```

   The loop bounds define the iteration space for loop induction variable `i`.

2. Two array references with array subscript (index) expressions of the form (true dependence)

   ```
      for i = LB, UB, 1
   R1:   X(a*i + c1) = ...        \\ write
   R2:      ... X(a*i + c2) ...  \\ read
      endfor
   ```

   where `a`, `c1`, and `c2` are integer constants, R1 and R2 are references to the same array, `i` is the loop induction variable, and `a` $\neq 0$.

Question: $\boxed{\text{Is there a true dependence between R1 and R2?}}$

Dependence Testing

There is a dependence between R1 and R2 **iff**

$$\exists i, i' : i \leq i' \;\; and \;\; (a * i + c_1) = (a * i' + c_2)$$

where i and i' are two iterations in the iteration space of the loop. This means that in both iterations, the same element of array `X` would be accessed.

So let's just solve the equation:

$$(a * i + c_1) = (a * i' + c_2) \Leftrightarrow$$

$$\frac{c_1 - c_2}{a} = i' - i = \Delta d$$

There is a dependence with distance Δd iff

1. Δd is an integer value and
2. UB - LB $\geq \Delta d \geq 0$

Dependence Testing Examples

1.
```
      for i = LB, UB, 1
R1:     X(i) = ...            \\ write
R2:         ... X(i - 2) ...  \\ read
      endfor
```

a=1, c_1=0, c_2=-2 $\Rightarrow \Delta d = 2$ (**dependence**)

2.
```
      for i = LB, UB, 1
R1:     X(2*i) = ...              \\ write
R2:         ... X(2*i - 1) ...  \\ read
      endfor
```

a=2, c_1=0, c_2=-1 $\Rightarrow \Delta d = \frac{1}{2}$ (**no dependence**)

Assume R1 executes before R2.

$\boxed{\text{Classification of dependences}}$:

- R1 is write, R2 is read $\Rightarrow$ **true** dependence
- R1 is read, R2 is write $\Rightarrow$ **anti** dependence
- R1 is write, R2 is write $\Rightarrow$ **output** dependence

Next Lecture

Things to do:

- Please see our OpenMP tutorial on our website
- More on dependence testing and loop optimizations